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DISCRIMINATORY *VERSUS* UNIFORM-PRICE AUCTIONS: AN EMPIRICAL ANALYSIS OF THE BRAZILIAN TREASURY AUCTIONS

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1.Leilões de títulos públicos; 2. Leilões de múltiplos objetos; 3. Leilões de preços múltiplos; 4. Leilões de preço uniforme

Aos meus alicerces: minha esposa Flávia, meus Pais e meus Irmãos, sem os quais qualquer esforço carece de sentido...

À minha avó, Zelinda, que faz jus ao nome como ela só.

Dedico

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“It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.”

Richard Feynman.

RESUMO

DISCRIMINATORY VERSUS UNIFORM-PRICE AUCTIONS: AN EMPIRICAL ANALYSIS OF THE BRAZILIAN TREASURY AUCTIONS

Este trabalho tem como objetivo determinar qual o melhor desenho para os leilões de títulos públicos da Secretaria do Tesouro Nacional em termos de geração de receita: leilões de preço uniforme ou leilões de preços múltiplos. Desde a década de 60, economistas têm debatido sobre o desenho ótimo para leilões de múltiplas unidades. Desde então, a literatura teórica tem desenvolvido argumentos a favor de ambos os tipos de leilão. Além disso, literatura empírica vem aplicando repetidas vezes métodos empíricos a dados de leilões de títulos públicos, também chegando a resultados ambíguos. Após realizar uma cuidadosa revisão da literatura, empregamos métodos empíricos a dados dos leilões do Tesouro Nacional, de modo a determinar o melhor formato de leilão para o Tesouro Nacional.

Palavras Chave: Leilões de títulos públicos, Leilões de múltiplos objetos, Leilões de preços múltiplos, Leilões de preço uniforme.

ABSTRACT

DISCRIMINATORY VERSUS UNIFORM-PRICE AUCTIONS: AN EMPIRICAL ANALYSIS OF THE BRAZILIAN TREASURY AUCTIONS

Our research aims to determine the best auction format for the Brazilian National Treasury auctions in terms of revenue: uniform-price or discriminatory auctions. Since the 1960's, economists have debated the optimal design of Treasury auctions. Since then, theoretical literature has developed arguments favoring either type of auctions. Furthermore, empirical literature has repeatedly applied empirical methods to data from Treasury auctions, also reaching ambiguous results. After conducting a careful literature review, we apply empirical methods to Brazilian Treasury auction data and determine the best auction format for the Brazilian Treasury.

Keywords: Treasury auctions, Multi-unit auctions, Discriminatory auctions, Uniform-price auctions.

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1 INTRODUCTION

1 INTRODUCTION

One of the main functions of a National Treasury is to raise funds for the Federal Government through debt issuance. There are a number of ways to issue government debt, but currently a great number of National Treasuries throughout the countries issue debt by way of competitive actions of government bonds. The Brazilian Treasury, for example, in 2013 alone raised approximately R$ 368 billion in competitive auctions, which corresponds to 7.6% of the country's GDP. These figures highlight how important it is to use appropriate auction designs.

A Treasury could use several auction formats to issue its debt, but in practice two sealed bid, multi-unit auction formats are dominant: the discriminatory and the uniform-price auction. The basic difference between these two types of auction is the pricing rule. In the discriminatory auction format, the price paid by any winning bid is the bid price itself. In the uniform-price format, every winning bid pays the same price (the market clearing price), usually the lowest winning bid. Naturally, this difference implies different incentives for the bidders and, therefore, influence their strategic behavior. Having said that, a question arises: which of these two auction formats is more advantageous from the Treasury's perspective?

This debate is old and still not finished. Early academic opinion supported that uniform-price auctions would result in greater revenue for the Treasury when compared to multiple price auctions. This early assessment was based on two types of arguments.

There is an informal argument that collusion is less likely in a uniform-price auction (FRIEDMAN, 1960). According to this argument, because of the severity of the winner's curse in discriminatory auctions, the market would be eventually limited to very specialized bidders. Consequently, bidding would become concentrated within a small number of bidders who would have incentives to collude.

The second argument is grounded on the theory of auctions for indivisible goods. According to this theory, in settings where the revenue equivalence theorem does not apply, second-price auctions generate more revenue on average then does a first-price auction (BUKHCHANDANI, HUANG, 1989; CHARI, 1992; MILGROM, 1989). Once again, the central issue is the winner's curse[1]. Because the winner's curse is less severe in second-price auctions, bidders end up bidding more aggressively so the second highest bid in a second-price auctions is higher on average than the highest bid in a first-price auction. Since the nature of the uniform price auction is very similar to the second-price auction and the discriminatory auction bear similarities with first-price auctions, the logic would also be valid for Treasury auctions.

Later theoretical studies contradicted the arguments above, showing that the theory used to reach such conclusions apply only to single-unit auctions. Treasury auctions are multi-unit auctions where the bidders can submit multiple bids. In this case, the single object auction theory (or even a multiple object theory where bidders have single unit demands) is too naive to capture all the nuances and to represent the whole set of possible equilibria. Back and Zender (1993), for example, developed a model for treasury multi-unit auctions, where they assumed bidders could submit a continuous demand schedule. The results of their studies showed the exact opposite of the prevalent opinion and indicated that discriminatory auctions, under certain assumptions, could yield more revenues than uniform-price auctions.

Despite the advances in multi-unit auction literature, no theoretical work has yet been able to model Treasury auctions satisfactorily. The challenge is to add complexity to the models while maintaining the problem still tractable and to deal with the multiplicity of equilibria. In the following section, we will describe some of these models and will discuss on the complexity and difficulties of modeling multi-unit auctions.

Due to the complexity of the subject and to the lack of a definitive recommendation in the theoretical literature, research turned to empirical analysis using data from Treasury auctions, often getting at opposing conclusions. These studies used data from a number of countries, such as Mexico, France, Canada,

[1] In common value auctions with incomplete information, the winner's curse is the tendency that the winner has to overpay. The winner may overpay or be "cursed" in one of two ways: 1) the winning bid exceeds the value of the auctioned asset such that the winner is worse off in absolute terms; or 2) the value of the asset is less than the bidder anticipated, so the bidder may still have a net gain but will be disappointed.

Turkey, the United States, among others. Although some work has been done to empirically test bidder behavior using data from the Brazilian Treasury auctions (GIELMAN, 2003; SILVA, 2003), to the best of our knowledge, no study has used Brazilian Treasury auction data and conducted empirical analysis aiming at ranking the auction formats in terms of revenue.

Our research contributes to this debate by analyzing empirical evidence of Brazilian Treasury auction data, and trying to establish which design is more advantageous for the Brazilian National Treasury, the uniform-price or the discriminatory auction. The auction data we use is bidder level data, which grant us a wide range of possibilities from an empirical perspective. There are several aspects that could be used to compare mechanisms and select a specific auction format[2], but in this study we will focus on seller's revenue both for their importance as a selection criterion and because there exists already extensive literature exploring this issue.

This study is organized as follows. Section 2 comprehends a careful review of the theoretical literature on the subject. We survey recent research on Treasury auctions, especially those that approach the subject from a theoretical point of view. Section 3 describes the auction environment put in place by the Brazilian Treasury, its rules and institutional particularities. Sections 4 and 5 describe and apply two different empirical approaches to Brazilian Treasury auction data, aiming at determine a revenue ranking for auction formats. Section 4 explores the policy experiment approach, while section 5 anlyses the structural model approach. Section 6 concludes and make suggestions for futures research.

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[2] Other criteria for mechanism selection could include efficiency, robustness, transaction costs, immunity to cheating and collusion and incentive for bidder participation.

2 LITERATURE REVIEW

2 LITERATURE REVIEW

2.1 IMPORTANT DEFINITIONS AND CONCEPTS

Before we begin with the literature review itself, we will introduce some concepts and definitions that will help the reader who is not familiar with auction theory to have a better understanding of the information conveyed in this section. By no means, however, do we intend to provide a complete introduction to auction theory[3]. Our aim is simply to introduce some concepts and definitions commonly used in the Treasury auctions literature.

An auction is fundamentally a price revelation mechanism. It is often used to trade goods in circumstances where there is incomplete information about the value of the goods traded. By value, we mean the maximum amount a buyer is willing to pay for an object. If, for example, a seller knew with certainty how buyers value a good, he could simply fix the price at the highest value assigned by any of the buyers. In this case, an auction would not only be unnecessary, but would also add costs to the transaction.

In practice, however, it is very common to find situations where there is at least some degree of uncertainty about how buyers and sellers value a good. In these situations, an auction represents a set of rules that describes the process that will establish the price of the good and to which buyer the good will be awarded. Obviously, sellers and potential buyers (bidders) behave strategically, both trying to maximize their objectives. Sellers will typically try to maximize the revenue earned in the transaction[4]. Bidders will compete with the seller and with each other, trying to maximize the chance of winning the auction and simultaneously minimize the price to be paid in case they actually win the auction.

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[3] We strongly recommend Vijay Krishna's book *Auction Theory* to anyone who wishes to study auction theory in a complete and systematic way.

[4] A seller may wish, as well, to guarantee that the result of the auction is an efficient one. We will return to this point later.

Combining the strategic aspect of the auction with the uncertainty concerning the value of the good, we have the key elements used to describe an auction: a game of incomplete information. This is the framework used in economic theory to conduct the analysis of auctions. This framework emerged in the seminal work of Vickrey (1961) and is still currently used by economists to model auctions.

The central issue when studying games of incomplete information is to determine the Bayesian Nash equilibrium (or equilibria) of the game or at least establish some properties of that equilibrium[5]. In the context of an auction, the Bayesian Nash equilibrium is an extension of the Nash equilibrium concept, used in games of complete information, and represent a set of strategies in which each bidder is maximizing his expected payoff, given the strategy played by the other bidders and given his beliefs about how other players value the object. In general, the Bayesian Nash equilibrium of an auction may be unique, multiple, or nonexistent. Henceforth, we simply will use the term equilibrium to refer to Bayesian Nash equilibrium, unless otherwise specified.

An equilibrium is efficient when it awards the auctioned object to the bidder who values it the most. From the seller's perspective, efficiency may be a desirable feature, especially when the seller is the government, but maximizing revenues is usually the priority. Efficiency and seller's revenue maximization do not always coincide. Although we will show some results where the ranking of auction formats according to revenues is linked to efficiency, this is not necessarily the case. It can only be proved that revenue and efficiency rankings coincide in specific environments (AUSUBEL, CRAMTON, 2002).

Important elements in auction analysis are the assumptions made about the environment of the case under study. These assumptions are restrictions or specification of special characteristics of the auctions that, in many cases, greatly

[5] Theory on Treasury auctions has focused on competitive (non-cooperative) equilibrium, despite persistent allegations that bidders in Treasury auctions often collude (BUKHCHANDANI and HUANG, 1993; UMLAUF, 1993). Collusion has received little attention in the theoretical literature of Treasury auctions.

influence the results of the analysis. These assumptions usually address characteristics of goods, bidders and payment rules.

The Treasury auction formats most commonly used are multi-unit, sealed-bid auctions. Multi-unit auctions comprehend the sale of multiple objects as opposed to single unit auctions, which involve the sale of one object. A sealed bid auction is a one-step procedure in which, as the name suggests, bidders submit sealed bids. Once all bids are submitted, the seller computes the result of the auction according to the pricing adopted. In open auctions, on the other hand, the procedure is conducted in multiple steps and bidders publicly announce their bids. Increasing bids are submitted until a maximum price is reached (in the case of single unit auctions), or until the total the total quantity submitted equals the supply (in the case of multi-unit auctions).

Other assumptions that should be applied to Treasury auction analysis vary from author to author, but we will describe next the main variants used in the literature. When reviewing the literature, we shall make explicit the assumptions used by each author.

As stated earlier, in an auction environment there is uncertainty about the value of the object being auctioned. Better defining assumptions on the nature of this uncertainty leads us to three different classes of model. Bidders never know with certainty (unless there is collusion) each other’s values for the auctioned object. If we assume that each bidder knows with certainty his own value for the object, we are dealing with a private value setting. In this setting, a bidder's value is unaffected by learning any other bidder's information.

Instead, we could assume the object’s value is unknown at the time of the auction even to the bidder himself. He may have only an estimate or some privately known signal that is correlated with the true value. In this setting, the interdependent value setting, information possessed by other bidders, if known, would affect the value that a particular bidder attaches to the object. The third class, the pure common value[6] setting, is a situation in which the actual value is the same for everyone, but bidders have different signals about what that value actually is.

In terms of the bidders' characteristics, the auction can be symmetric, if bidders' private values or signals are drawn from a common distribution or asymmetric otherwise. Bidders can also be risk neutral or risk averse. Risk neutral bidders seek to maximize their expected profits. Risk averse bidders seek to maximize their expected utility, which is some increasing, concave function of their profits. Asymmetry can stem also from the fact that bidders have different degrees of risk aversion.

Concerning payment rules, there is a great variety of formats. In Treasury auctions, as mentioned in the introduction, the uniform-price and the discriminatory auctions are widely dominant. Next, we detail these two formats, but some single unit auction formats are also described in Table 1 because the theoretical literature often refers to them.

Table 1. Commonly used single unit auction formats

English auction The English auction is an open single unit auction, in which prices are progressively raised, either by the auctioneer or with the bidders placing their bids, until there is only one interested bidder. There are adaptations of this format for multi-unit auctions as well.	**Dutch auction** In the Dutch, auction the auctioneer begins by calling out a price high enough so that presumably no bidder is interested in buying the object at that price. This price is gradually lowered until some bidder indicates his interest. The object is then sold to this bidder at the given price.
First-price auction The first-price auction is a sealed-bid single unit auction. In this auction, the bidders submit sealed bids. The object is awarded to the highest bidder and he pays the price he bids.	**Second-price auction** The second-price auction is a sealed-bid single unit auction. In this auction, the bidders submit sealed bids. The object is awarded to the highest bidder but the winner pays the bid of the second-highest bidder.

The discriminatory auction is a sealed-bid multi-unit auction. In terms of procedure, it is a natural extension of the first-price auction for multiple objects.

[6] In the literature, the terms *common values* and *interdependent values* may have different meanings from the ones used here. We follow the definition in Krishna (2002).

Suppose a seller announces it wants to auction K identical objects. The procedure begins with the bidders submitting sealed bids. Only this time each bidder may submit multiple bids, which form a quantity-price schedule. After all bids are received, the seller adds up the quantities bid, starting at the highest price bid and moving down, until the sum hits the total supply K. The bids that compose this sum are the winning bids. Observe that, since a bidder is allowed to submit multiple bids, some of his bids may be awarded, while other bids may lose the auction. Each bidder then receives the quantities informed in his winning bids (if any) and pays the price he stated in these bids.

The uniform-price auction is very similar to the discriminatory auction. In fact, the procedure is identical until the winning bids are determined. The only difference is the price paid by the winning bids. In the uniform-price auction, each bidder also receives the quantities informed in his winning bids (if any) but every bidder pays the exact same price for his winning bids. This price, the market-clearing price, can be defined as the lowest winning bid or the highest losing bid. Because, the winning bids are not filled at the prices bid, the uniform-price auction is considered a natural extension of the second-price auction for multiple objects.

One last topic has to be addressed before we proceed with the literature review: the Revenue Equivalence Theorem (RET). The RET is a central result in auction theory and can be stated as following:

Suppose that values are independently and identically distributed and all bidders are risk neutral. Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue to the seller.

In other words, under certain assumptions, the seller can obtain no gains in expected revenue by switching auction formats. Under these assumptions, every auction format is equivalent from the seller’s expected revenue perspective. This is a very strong result and apparently renders our research pointless. If the RET was valid for Treasury auctions, discriminatory or uniform-price auctions would be equivalent in terms of expected revenue and the Treasury would have to use a different criterion to choose between the two.

One can argue, however, that the RET is not valid for Treasury auctions. A modified version of the RET is valid for multi-unit auctions when bidders want one object only (single unit demand). However, this is not the case of Treasury auctions, where bidders have multi-unit demands. In multi-unit auctions with multi-unit demands, the weak RET require that the auction formats be allocationally equivalent, which, as we will see, is not the case of discriminatory and uniform-price auctions. It is only in very specific cases of multi-unit auctions with multi-unit demands that one can argue on *a priori* grounds that some form of the RET still holds, and these specific cases do not properly describe the Treasury auction environment (AUSUBEL, CRAMTON, 1997; ENGELBRECHT-WIGGANS, KAHN, 1998).

2.2 SINGLE UNIT AND SINGLE UNIT DEMAND THEORETICAL LITERATURE

When the debate on the selection of an appropriate auction format for Treasury securities began, back in the early 1960s, auction theory was a newborn field in economics. In fact, Milton Friedman initiated the debate with an informal argument. He argued that, because of the severity of the winner's curse in discriminatory auctions, the market would be eventually limited to very specialized bidders (FREIDMAN, 1960). Consequently, bidding would become concentrated within a small number of bidders who would have incentives to collude.

Later, other scholars attempted to use formal arguments to address this debate. Initially, however, since there was very little development in multi-unit auction theory, these efforts were centered on deriving analogies from single unit auction theory to Treasury auctions. This imperfect analogy, which shall be criticized later, was often used in discussions of Treasury auctions (BUKHCHANDANI, HUANG, 1989; CHARI, 1992; MILGROM, 1989; NANDI, 1997).

The pioneer in auction theory, and the first author to define a ranking among auction formats was Vickrey (1961). In this influential work, Vickrey derived some important results. He was the first to prove a revenue equivalence result, when comparing the English and the Dutch auctions, for the case of two risk neutral, symmetric bidders with private independent values drawn from a uniform distribution[7]. His comparisons of the English and Dutch auctions went further. He establishes that, under symmetry, both auction formats could be efficient. If the symmetry is abandoned, however, the English auction could still produce efficient outcomes, while the Dutch auction, in general does not. It is a dominant strategy for the bidders, under the English auction, to bid their true value.

Vickrey (1961) also extends his analysis for single unit sealed-bid auctions. It argues that, under the private independent model, the first-price auction is isomorphic to the Dutch auction, and the second-price auction is isomorphic to the English auction. Consequently, the second-price auction also induces bidders to bid their true value, which implies efficient outcomes. Under a first-price auction, bidders shade[8] their bids, but efficiency is still possible in symmetric cases. Furthermore, all four auctions formats yield the same expected revenue to the seller.

It also suggests another possible gain of the English and second-price auctions over the Dutch and first-price auctions. Because bidders bid sincerely in the English and second-price auctions, bidders who participate in the auctions are only concerned with their own appraisal of the object and not with an estimate of the value that others place on it. The same thing does not apply to the Dutch and first-price auctions. Therefore, participating in the latter auction formats would imply greater information gathering costs for the bidders. This greater cost would eventually reduce the number of participants in these auctions, thus reducing the expected seller's revenue.

Most of Vickrey's results focuses on single unit auctions. Nevertheless, he also consideres multi-unit auctions in his work, and suggestes that uniform-price auctions could carry some of the advantages of the second-price auctions. However, he made clear that this result applies only to cases where each bidder is interested in at most a single unit. He also warned:

[7] Myerson (1981) only proved the general result several years later.

[8] Bid shading is a term that represents the practice in which bidders submit a bid below the value they attribute to the goods being auctioned.

It is not possible to consider a buyer wanting up to two units as merely an aggregation of two single-unit buyers: combining the two buyers into one introduces a built-in collusion and community of interest, and the bid offered for the second unit will be influenced by the possible effect of this bid on the price to be paid for the first…

Vickrey's analysis is restricted to the case of risk neutral, symmetric bidders with private independent values. Other authors undertook the task of analyzing these auctions with different assumptions.

Milgrom and Weber (1982) develo a general model for symmetric, risk neutral bidders with interdependent values. Their first result is that the Dutch and first-price auctions are still strategically equivalent in the general model, just as they were in the private independent model. The English and the second-price auctions, though, are no longer equivalent. Hence, the revenue equivalence result valid in Vickrey's model no longer applies. The authors show that, when bidder's values are correlated, the four types of auctions can be ranked as follows, from highest to lowest seller's expected revenue: (1) English auction, (2) Second-price auction, (3) first-price and Dutch auctions (tied).

They also explore a model with private independent values and risk aversion, and show that the first-price auction leads to higher prices than the second-price auction. If the model includes both interdependent values and risk aversion, the first and second-price auctions cannot be generally ranked by their expected revenues.

Similarly to Vickrey, Milgrom and Weber also acknowledge that one should have caution when trying to project these results to the multi-unit case:

Another issue that has received little attention in the bidding literature concerns auctions for shares of a divisible object[9]*. Recent studies indicate that such auctions involve a host of new problems that require careful analysis.*

---

[9] The divisible good auction is a better approximation of a multi-unit auction than a single unit auction. We shall return to this point when reviewing the multi-unit auction literature.

Despite these words of caution, many authors and policymakers have tried to establish direct analogies from single unit auctions or from multi-unit auctions with single unit demand to argue in favor of the uniform-price format for Treasury auctions. Milgrom himself in a later study (MILGROM, 1989) suggested that the US Treasury might do better by adopting a uniform-price auction in place of the discriminatory auction. His suggestion is based on a model in which multiple goods are auctioned, but each bidder wants only one unit.

Bikhchandani and Huang (1989) make a similar assessment. The authors develop a model of Treasury auctions where risk averse bidders, in a common value setting, bid for k identical objects. Each bidders, however, demands (or is allowed) at most a single unit of the object. As an additional feature, their model includes the existence of a resale market, so that bidders' strategies in the auctions would influence the securities resale price. Although the authors admit that the single unit demand is a simplification, they affirm that the uniform-price auction result in greater expected revenues for the auctioneer, when compared to discriminatory auctions. They also find that, in both formats, the existence of a good flow of information between primary and resale markets could increase seller's revenue. The rationale is that if bidders in the primary market win the auction at a low price, participants in the resale market will use this information and revise their own value estimate. Therefore, primary bidders have an incentive to bid aggressively to signal a high value.

Chari and Weber (1992) also state that a switch to either an English auction or a uniform-price auction for US Treasury debt is likely to raise Treasury revenues and reduce excessive resources devoted to information gathering. The argument is once again based on an analogy with single unit auctions. The authors argue that, in second-price auctions, risk-neutral bidders bid their true value. Since the uniform-price auction is a natural extension of the second-price auctions, bidders would face similar incentives in uniform-price auctions. This implies that in uniform-price auctions bid shading would be less severe than in discriminatory auctions and the incentives to acquire information about other bidders' values would be smaller. The authors recognize that matters are more complicated when bidders submit demand schedules, but they argue that the economic logic of the arguments for the single unit environment seems likely to carry over.

In short, part of the academic literature analyzed Treasury auctions through an analogy of single unit or single unit demand theory. This literature tends to view the uniform-price auction as superior to the discriminatory format (at least under risk neutrality), because the uniform-price auction is likely to inherit some advantageous features of the second-price auction. This analogy, however, is imperfect and, as we shall now discuss, leads us to conclusions that are not necessarily true.

2.3 MULTI-UNIT AUCTION LITERATURE

The uniform-price auction procedure reduces to a second price auction when there is only one unit for sale. Thus, the uniform-price auction appears to be a natural extension of the second-price auction to the multi-unit case. Nevertheless, this is not the case and the uniform-price auction does not share some important properties with the second-price auction.

Krishna (2002) explores this point by using a model where bidders are symmetric and risk neutral and values are private and independently distributed. In his model, there are K identical objects for sale and the bidders compete for all the objects (multi-unit demand). The author does not explicitly calculate equilibrium strategies for the uniform-price auction, but derive some important properties that any such equilibrium must have.

Under the stated assumptions, Krishna deduces that in every undominated equilibrium of the uniform-price auction, the bid on the first unit is equal to the true value of the first unit, while the bids on other units are lower than their respective marginal values. In other words, in equilibrium, bidders will not shade their bids on the first unit but they will shade their bids on all remaining units. Bidders do not bid truthfull as an analogy of the second-price auction would imply.

As a corollary of this property, the author obtains another property: every undominated equilibrium of the uniform-price auction is inefficient. Efficiency, another important property of the second-price auction is not exhibited by uniform-price auctions. Therefore, the uniform-price auction is not an extension of the second-price auction for the multi-unit case. In fact, Krishna argues that the proper extension of the second-price auction for the multi-unit case is a third type of auction, known as the Vickrey auction.

It is noteworthy that the inefficiency of the uniform-price auction does not emerge from the fact that multiple units are auctioned, but because bidders have multi-unit demands, that is, bidders submit a demand schedule. The ability to submit demand schedules grants an important strategic advantage to bidders, especially in uniform-price auctions. This ability produces great changes in the possible existing equilibria.

Although Vickrey, in 1961, already alerted to this fact, it took a long time until researchers addressed this question. One of the earlier works that did it was Wilson (1979), which served as the backbone for several posterior studies. Wilson models a share auction, where there is an object of which shares are to be sold to several bidders. More importantly, each bidder submits a sealed schedule of prices bid for varying fractional shares of the object. The demand schedules submitted by the bidders are continuous functions. His model shows that in a share auction, the seller's expected revenue is lower than if the object was sold as an indivisible auction in a single-unit auction. It is a theoretical proof of the market power that multi-unit demand can hand over to bidders.

Back and Zender (1993) extended Wilson's work focusing on Treasury auctions. In their model, a seller wishes to sell a fixed quantity Q of a perfectly divisible good to n bidders. Bidders are symmetric, risk neutral and operate under a pure common value assumption. The authors claim that bidders buying multiple units are concerned with marginal cost rather than price. Marginal cost is endogenous, being determined by the demand schedules submitted. Because of that, by submitting very steep demand curves in a uniform-price auction, bidders can make marginal cost very high for other bidders, inhibiting competition. Therefore, a "collusive" outcome is enforced in a non-cooperative equilibrium. In the discriminatory auction, however, such collusive equilibrium is not likely to hold.

Back and Zender use a simple example to illustrate the intuition behind this argument:

Suppose $10 billion of notes are to be sold and there are three bidders. Suppose each bidder knows that the yield in the after-market will be 5 percent. […] Consider the following strategies: each bidder bids for $3333 million at 6% and bids for $6667 million at 20%. Given a uniform-price format, the entire $10 billion would be *sold at 20%. […] The point we wish to emphasize is that this "collusion" on the part of the bidders is consistent with self-interest. […] Specifically, each bidder is getting \$3333 $\frac{1}{2}$ million at 20 percent. […] To increase the quantity above \$3334 million will cause the yield to drop from 20 to 6 percent or bellow, which would be certainly not profitable, so adhering to the collusive arrangement is optimal.*

[…]

The "collusive" equilibrium unravels in a discriminatory auction. If other bidders are bidding in the way described above, then each bidder will find it optimal to bid for the entire quantity at, say, 19.99 percent. […] However, if everyone does this, than each will find it optimal to bid for the entire quantity at 19.98 percent (thereby capturing the entire \$10 billion), and so forth. In fact, in any pure strategy equilibrium of a discriminatory auction, the yield will be 5 percent on the entire issue. Therefore, a discriminatory auction is an efficient mechanism for selling securities in this situation.

Back and Zender's model formalizes the intuition conveyed in this example and identifies some classes of equilibria in which uniform-price auctions are worse than discriminatory auctions, in terms of seller's revenue. These equilibria resist even if the fixed supply assumption is replaced by a random supply, which is an important feature, given the role played by noncompetitive[10] bids in Treasury auctions.

Wang and Zender (1996) further explore this model. They retain the same basic assumptions of Back and Zender's models, but include noncompetitive bidding (random supply) and examine the case where bidders are risk averse. They find that if the numbers of competitive bidders and the average level of noncompetitive demand are sufficiently large, there exist equilibria of the uniform-price auction that

[10] The noncompetitive bidder agrees to purchase a certain number of securities at the average price established in the auction. The noncompetitive bid process allows smaller investors to buy Treasury securities in a market that would otherwise be dominated by large institutional investors. Although noncompetitive bidders do not participate in the price determination process, they deplete part of the auction supply.

yield a larger expected revenue when compared to the discriminatory auction. This result indicate that risk neutrality plays an important role in Back and Zender's model.

In some countries, the Treasury has the right to choose the quantity of securities to be auctioned after the bidders submit their bids[11]. In this scenario, the Treasury could endogenize the quantity to maximize its revenue, a feature that none of the studies we reviewed so far have investigated. Back and Zender return to their model (Back and Zender, 2001) and look into this problem. The new model is similar to Back and Zender's (1993), except that the seller choses the actual quantity to be sold (restricted to an interval $[0, \bar{Q}]$) after he observes the demand schedules submitted by the bidders. The seller chooses the quantity that maximizes his expected revenue. The authors find that, in equilibrium, the seller's right to restrict the supply *ex post* is not used. However, it places a limit on the steepness of the aggregate demand curve and reduces equilibrium underpricing. The seller's faculty of choosing quantity *ex post* can thus restrict the bad equilibria that may occur in uniform-price auctions with multi-unit demand.

The findings of the studies we just reviewed (Wilson, Back and Zender and Wang and Zender) are very important ones and shaped the way researchers approached Treasury auctions thereafter. Nevertheless, two limitations are common to those works. First, they all explore models where bidders have pure common value. In such models, efficiency is not an issue and every allocation is equally efficient. In addition, their revenue comparisons are restricted to analysis of particular classes of equilibria. Since there might exist other equilibria, their results lack generality.

Ausubel and Cramton (2002) deal with both limitations. They adopt a share auction framework, similar to Back and Zender's, but use an interdependent value setting and extend their analysis to the entire set of equilibria. For most of their work, however, they use a simplifying assumption that bidders' demand schedule are flat[12].

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[11] For example, the Brazilian Treasury has the right to reduce, partially or totally, the auction's announced supply after receiving the bids.

[12] Flat demand schedules refer to a bidder's demand schedules in which marginal values are constant.

Regarding efficiency, Ausubel and Cramton argue that the outcomes from uniform-price auction are not efficient due to the existence of bid shading (demand reduction) and differential bid shading over quantity. Since in the uniform-price auction bid on later units can influence the price a bidder will pay, bidders try to shade more for later units in order to pay less on the earlier units. In the discriminatory price auction this incentive does not exist, so bidders tend to shade bids by similar amounts and an efficient outcome is more likely to occur.

Regarding revenue comparison, their main finding is that the revenue rankings of the uniform-price and discriminatory auctions are inherently ambiguous. Specifications of demand determine which auction format dominates the other. They also suggest several theorems about the relationship between efficiency and the revenue maximization. With symmetric bidders and flat demands, the revenue maximization and efficiency coincide, that is, the revenue maximizing auction awards all quantity to the buyers who value them the most. However, this does not generalize to the case of downward sloping demand curves. Their conclusion is that, in general, there is a conflict between revenue maximization and efficiency with downward sloping demands.

The ambiguity in rankings of the two auction formats apparently can only be reduced in highly stylized models. Wostek, Weretka and Pycia (2009) develop a model with no private information and restrict their analysis to a specific class of equilibria, the linear equilibria. The choice of linear equilibrium can be justified empirically, since some empirical studies[13] have shown that linear bid schedules provide an excellent fit for bid schedules observed in auction data. Furthermore, much of the analytical difficulties associated with the study of multi-unit auction derive from the large size of the strategy space. Restricting the analysis to linear equilibria reduces that space to two dimensions: the slopes and the intercept of bidders' schedules.

The authors show that the linear Bayesian Nash equilibrium, when it exists, is unique for both auction formats, in asymmetric as well as symmetric settings, which enables them to establish consistent rankings. In terms of expected revenue,

[13] Hortaçsu (2002) and Hortaçsu and Puller (2008).

the discriminatory auction dominates the uniform-price auction for a given market size. This discriminatory auction's dominance weakens as the number of bidders increase. In the limit case, both format are revenue equivalent. Therefore, in large markets, a risk neutral auctioneer should be indifferent among auction formats. Again, in the limit case, while equivalent in *ex ante* revenues, the formats are not equivalent *ex-post*: the revenue in the uniform-price auction stochastically dominates that in the discriminatory auction in the second-order sense and should be preferred by a risk averse auctioneer. The following table summarizes the revenue rankings:

Table 2. Revenue rankings in Wostek, Weretka and Pycia (2009)

	Auctioneer	
Auction	Risk Neutral	Risk Averse
Small	$DPA \succ UPA$	*It depends*
Large	$DPA \sim UPA$	$DPA \prec UPA$

Source: Wostek, Weretka and Pycia (2009)

At this point, the reader is probably being led to conclude that discriminatory auctions are (weakly) superior to uniform-price auctions. There is a class of equilibria in the uniform-price auction in which seemingly collusive outcomes may produce arbitrary underpricing. In addition, the uniform-price auction is inefficient in environments in which the discriminatory is not and their revenue rankings is inherently ambiguous in a general analysis. In more restricted settings, where a sharp ranking can be constructed, the discriminatory auction dominates the uniform-price auction under more plausible scenarios. These findings combined could imply that a Treasury would be better off by adopting a discriminatory format. However, this is not true in general.

Regarding underpricing in uniform-price auctions, recent studies show how the auctioneer could eliminate at least some of the undesirable equilibria by slightly modifying the uniform-price auction. We have already learned from Back and Zender (2001) that bidders' market power can be reduced if supply can be adjusted by the seller after the bid schedules are observed. McAdams (2007) obtains similar results. Licalzi and Pavan (2005) show, additionally, that similar results may be achieved when the seller precommits and declares the supply schedule before observing the bid schedules. They recognize that this precommitment lead to the risk of losing control on the quantity sold, but argue that a perfectly elastic supply may be appropriate when the cost of issuing debt in a variable supply is small compared to the benefit of controlling the interest rate in the primary market.

Other studies discovered that some conditions present in real world auctions can reduce underpricing in uniform-price auctions. That is, in a framework that better represents Treasury auctions, underpricing in uniform-price auctions may be less severe than previous theory might indicate.

Kremer and Nyborg (2004) is an example of such literature. The authors use the same basic model of Wilson (1979) and Back and Zender (1993). However, instead of assuming that demand schedules are continuous functions, they assume that demand schedules are discrete. This new feature better represents reality, since bidders are typically asked to submit price-quantity pairs, not continuous functions. It may seem a minor adjustment, but imposing discreteness over bids in this model reduces underpricing equilibria. The reason is that discreteness makes the marginal residual supply significant, while in continuous demand functions it measures zero. This creates price competition over the marginal units and reduces underpricing.

The authors also investigate the effects of tick size and quantity multiple, which are also very common in practice. Tick size and quantity multiple are, respectively, the smallest increment by which a bidder can alter the price and the quantity in their demand schedule. Kremer and Nyborg show that underpricing in uniform-price auctions can be made arbitrarily small by an appropriate choice of tick size and quantity multiple.

Kastl (2012) also explores the effect of discreteness over equilibria in divisible good auctions. Similarly to Kremer and Nyborg, he starts with the basic share auction Wilson/Back and Zender framework and assumes that bidders are restricted to use step functions. Additionally, the author assumes that there is an upper bound on the allowed number of bid points, which is common in practice. He finds that the revenue of the uniform-price auction in which bidders bid truthfully their values does not constitute an upper bound on the *ex post* revenue of the uniform-price auctions. Bidders may find optimal to submit bids that are higher than their marginal valuations in uniform-price auctions. The author also finds that the loss from using only k steps, rather than a continuous bid function, is of the order of $1/_{k^2}$. Both findings have significant empirical relevance.

Beyond the fact that bad equilibria in uniform-price auction may not be as severe as continuous demand schedule theory might suggest, uniform pricing has other desirable characteristics: it is easy to understand, it is fair in the sense that an equal price is paid by all winning bidders, if market power is not exercised it is efficient and strategically simple, it requires less information gathering and in the presence of market power uniform pricing favors smaller bidders.

Friedman's argument that uniform-price auctions foster entry of participants should also be taken into account. Ausubel and Cramton (2002) show that the argument is likely to be true. Besides the usual argument that uniform pricing reduces the penalty for guessing wrong, their model show that uniform pricing also creates an incentive for large bidders to make room for smaller bidders. Wostek, Weretka and Pycia (2009) extend their own static linear equilibria model, including a dynamic game where bidders, after learning the outcome of the auction, sequentially choose whether to join an auction or not. They find that the uniform-price auction format encourages more entry than the discriminatory auction. Even if the difference is quantitatively small, the difference in bidder participation may reverse the revenue rankings in table 2.

In conclusion, when it comes to revenue comparisons between discriminatory and uniform-price auctions, multi-unit theoretical literature has achieved great advances. However, it is not yet capable of providing a definitive recommendation whether the criterion is revenue maximization or allocation efficiency. The bidders' ability to submit multiple demands significantly builds up the analytical complexity of these auctions. Multiplicity of equilibria and ambiguity of efficiency and revenues rankings, even under rather simplifying assumptions, seem to be serious obstacles to a definitive policy recommendation. It appears that, at least for the time being, the words of Ausubel and Cramton are valid: "Thus, if the seller is constrained to select between the discriminatory and the uniform-price auction, the choice ought to be viewed as an empirical question".

2.4 EMPIRICAL LITERATURE

The lack of a definitive recommendation in the theoretical literature lead research to turn to empirical analysis in search of answers and insights that would ultimately help establish the best auction format for Treasury auctions, discriminatory or uniform-price. As we will observe, empirical analysis also produces ambiguous results. Nevertheless, progress has been made and studies have been conducted with data from a number of countries, such as Finland, Mexico, Sweden, Turkey, the United States, among others. These studies are generally classified under two distinct empirical approaches: policy experiments and structural models.

Earlier works usually fall under the policy experiments category. These studies take advantage of situations in which a Treasury switches the auction format at a certain point in time, thus providing a “natural” experiment that allows a direct comparison of the two formats. Researchers then investigate the coefficient of a dummy variable for the change of the auction mechanism in a regression model in which the dependent variable is one indicating bidders’ markup or profit obtained from the auction. Profits are obtained comparing the difference between the auction price and the price in the resale or when-issued markets, which is assumed as the true value of the security auctioned. The main hypothesis is that if the coefficient is significant, then this indicates that the switch of the auction format affects the bidders’ profit and, consequently, seller’s revenue. Additionally, the sign of the dummy’s coefficient indicates which auction format is preferable regarding revenue.

This approach is relatively easy to implement and establishes a direct comparison between the two auction formats in terms of revenue. The results are straightforward to interpret and communicate to government officials. However, the policy experiment approach carries a number of disadvantages likewise. First, it requires that the Treasury auction under analysis has been subject to a format switch, which does not happen very often. Another drawback of this approach is that its validity relies on the assumption that the researcher can control for all factors that may have changed between the end of the auction and the beggining of the trade in the resale market. Finally, this approach analyses aggregate auction data. On one hand, these are usually easier to obtain. On the other hand, it cannot capture important strategic behavior occurring in bidder level data and, therefore, fail to deliver richer empirical information.

The structural model approach employs a completely different technique and has usually been applied in more recent research. Here, the goal is to derive, by solving a certain theoretical model, the bidders' optimality condition that maps the observed bid data to the unobserved variables of interest, such as the distribution of marginal valuations. These conditions are then used to estimate or recover the unobserved variables.

The estimation of model primitives enables the researcher to construct counterfactual comparisons in which the *ex ante* conditions are held fixed, hence eliminating the main fragility of policy experiments. Furthermore, the structural model approach utilizes bidder level data, which permits to compare the auction formats in terms of their distributional aspects (efficiency). On the downside of structural models are the lack of closed theoretical solutions for multi-unit auctions, the dependence of a specific theoretical model and the difficulty in obtaining bidder level data.

Policy Experiments

One of the first studies that use the policy experiment approach to compare the revenues generated by the two auction formats is Umlauf (1993). He analyses the Mexican Treasury auctions, more specifically Mexican 30 day T-bill auctions. In mid-1990, the Mexican Treasury switched its T-bill auction format from discriminatory to uniform-price. This provided the opportunity to utilize the policy experiment approach and compare the revenue generated by both formats. In his sample, 181 auctions are discriminatory and 26 use uniform pricing. Umlauf finds that when the Mexican Treasury switched to the uniform-price auction, bidders' profits were eliminated and seller's revenues increased. The author also argues that this result suggests, although does not prove, the existence of collusion among the largest bidders in Mexican Treasury auctions[14].

[14] Umlauf's initial suspicion of collusion in Mexican Treasury auctions was raised during conversations with market participants.

Simon (1994) uses the results of auctions for 15-30 year maturity Treasury bonds in the United States. The data covered the period from 1973 to 1976, in which 6 single price auctions and 10 multiple price auctions were held. He applies the policy experiment approach using, instead of profits as the dependent variable, bidders' "markup" rate measured by the average winning rates minus when-issued market. One must be aware that higher "markup" in yields means actually "markdown" in terms of bid price. In his regression, the dummy coefficient for uniform-price auction is 0.08, which implies that "markup" is 8 basis points higher in the uniform-price auction than in the discriminatory auction. Therefore, the author argues that the revenue from the uniform-price auction decreased compared to the discriminatory auction, a result contrary to Umlauf's.

Subsequently, Nyborg and Sundaresan (1996) and Malvey and Archibald (1998) also use U.S. data and compare both auction formats in terms of revenues. Either studies are unable to find any statistically significant difference in the revenues generated by the two auction formats, contrary to the result found by Simon (1994).

Goldreich (2007) also studies US Treasury auctions with a more recent database that comprises 105 discriminatory and 178 uniform-price auctions between June 1991 and December 2000. He finds that, in his data set, both auction formats were subject to bid shading (underpricing) but in the uniform-price auctions the magnitude of the underpricing was smaller on average. He also constructs a model of multi-unit common values auction with unit demand and shows that the magnitude of the underpricing he estimated is consistent with this model[15].

Structural model approach

The first work to apply the structural model approach to compare Treasury auction formats is Heller and Lengwiler (1998). They analyze the Swiss Treasury market using the theoretical model developed by Nautz (1995). In Nautz's model, however, bidders are assumed price takers and so important bidders' strategic

[15] He explicitly makes this point in his article, even though most of the results regarding this issue were statistically insignificant.

aspects are neglected. Since structural models are highly dependent on the theoretical model adopted, this calls into question the reliability of their results. Nevertheless, the authors concluded that uniform-price auctions generate higher revenue for the Swiss Treasury than discriminatory auctions.

The real breakthrough in the structural model approach came with Hortaçsu (2002), which establishes great advances in both the theoretical modeling and the empirical methodology. He constructs a model with symmetric, risk neutral bidders with independent private values and random supply based on Wilson (1979)'s setup and derives the optimal condition for the discriminatory auction. Additionally, he extends Guerre, Perrigne and Vuong (2000)'s method of nonparametric estimation to a multi-unit demand case to estimate bidders' true valuations. He applies this method to the Turkish 13-week Treasury bill auctions, covering 25 discriminatory auctions held between October 1991 and October 1993. Hortaçsu finds that revenue equivalence between a discriminatory auction and the "best case" uniform-price auction cannot be rejected. This study was later revised (Hortaçsu and McAdams, 2010) and added to the conclusion that the discriminatory auctions analyzed were close to fully efficient.

Férvrier, Préget and Visser (2004) also propose a structural econometric method for the empirical study of Wilson's share auction model. While Hortaçsu (2002) estimates a nonparametric model assuming private values, Férvrier, Préget and Visser estimate a parametric model with common values. The authors apply the method to all French Treasury's bond and note auctions held in 1995. Their counterfactual comparison shows that Treasury's revenue in discriminatory auctions is higher than in uniform-price auction.

Castellanos and Oviedo (2005) apply the structural model proposed by Férvrier, Préget and Visser (2004) to Mexican Treasury auctions. They analyze Mexican T-bill auctions carried out from January 2001 to April 2002. Their results confirm Umlauf (1993)'s policy experiment and show that the uniform-price auction yields more revenues to the Treasury than the discriminatory auction.

Armantier and Sbaï (2006) analyze 118 French Treasury's bond and note discriminatory auctions, which took place between May 1998 and December 2000. They develop a common value model largely inspired in Wang and Zender (2002) that differs from the ones used in prior studies in two important aspects. Their model accounts for risk aversion and is developed so that it is possible to test for informational and risk aversion asymmetries. Their conclusion is that both the French Treasury and the auctions' participants would have benefited if the auctions in their sample had been conducted under the uniform-price format instead of the discriminatory format[16]. This result opposes the conclusions claimed in Férvrier, Préget and Visser (2004), conducted under risk neutrality and symmetry assumptions. The authors argue that the "*failure to account for potential risk aversion and asymmetries in Treasury auctions may therefore have serious consequences, as it may lead us to conclude in favor of the incorrect auction format*".

The relevance of this assertion is confirmed in Armantier and Lafhel (2009), who apply the method developed in Armantier and Sbaï (2006) to a sample of Canadian government securities auctions, containing 100 discriminatory auctions held by the Canadian Treasury from October 1998 to September 2005. In contrast to what Armantier and Sbaï (2006) detected in French Treasury auctions, the authors could not identify any major asymmetry across participants at Canadian government securities auctions. They also find that, in this case, the discriminatory format is superior to the uniform-price format in terms of the revenues they generate, thus endorsing the importance of the symmetry assumption to the structural model approach.

Kang and Puller (2008) follow the approach of Hortaçsu (2002) and extend his methodology to uniform-price auctions. They apply the methodology to Korean Treasury auctions and analyze 30 auctions (10 discriminatory and 20 uniform-price auctions) that took place from September 1999 to April 2002. They find that Korean Treasury's discriminatory auctions lead to statistically higher expected revenue when compared to uniform-price auctions. They also show that the discriminatory format better allocates the Korean Treasury bills to the highest valuation bidders, in other words, the discriminatory format is more efficient than the uniform pricing format. The

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[16] Armantier and Sbaï (2007) later include in their counterfactual comparisons two additional auction formats (the "Spanish" and "k^{th} average price" auctions) and rank the four auction format in terms of seller's revenue as follows: "k^{th} average price" auction > uniform-price auction > "Spanish" auction > discriminatory auction.

differences in both revenues and efficiency are small, and the authors attribute this fact to a highly competitive market that mitigates the strategic differences between the two auction formats.

Another study that follows Hortaçsu (2002) is Kastl (2011). Kastl extends Hortaçsu's model and estimation method to account explicitly for an important feature: equilibrium strategies are step functions rather than continuous, differentiable ones. As we have already discussed in the theoretical literature review, when bidders are restricted to a discrete, finite equilibrium step function, they may submit bids higher than their marginal values for some units. Consequently, comparisons as the ones conducted by Hortaçsu (2002) may underestimate the revenue arising from the uniform-price auction and, therefore, bias the results towards the discriminatory auction. The author then proposes a new method to evaluate the performance of the adopted auction format, based on estimating the effectiveness of values extraction and the efficiency of the allocation. This method is applied to Czech Treasury's uniform price auction data and concludes that the uniform price auction performs well, allocating efficiently and failing to extract at most 0.03% worth of expected bidders' surplus. He attributes the excellent performance of the mechanism to the flexibility of the auctioneer to adjust supply after receiving the bids.

Table 3. Summary of reviewed Policy Experiment studies

Study	Object	Results (revenue)
Umlauf (1993)	Mexican Treasury bill auctions	UP auction > Discriminatory auctions
Simon (1994)	US Treasury bond auctions	UP auction < Discriminatory auctions
Nyborg and Sundaresan (1996)	US Treasury bond auctions	Statistically insignificant differences
Malvey and Archibald (1998)	US Treasury bond auctions	Statistically insignificant differences
Goldreich (2007)	US Treasury bond auctions	UP auction > Discriminatory auctions

Table 4. Summary of reviewed Structural Model studies

Study	Object	Results (revenue)
Hortaçsu (2002/2010)	Turkish Treasury bill auctions	Does not reject revenue equivalence
Férvrier, Préget and Visser (2004)	French Treasury bond auctions	UP auction < Discriminatory auctions
Castellanos and Oviedo (2005)	Mexican Treasury bill auctions	UP auction > Discriminatory auctions
Armantier and Sbaï (2006)	French Treasury bond auctions	UP auction > Discriminatory auctions
Armantier and Lafhel (2009)	Canadian Government bond auctions	UP auction < Discriminatory auctions
Kang and Puller (2008)	Korean Treasury bond auctions	UP auction < Discriminatory auctions
Kastl (2011)	Czech Republic Treasury bond auctions	UP auction extracts almost all bidders' surplus

3 BRAZILIAN TREASURY AUCTIONS

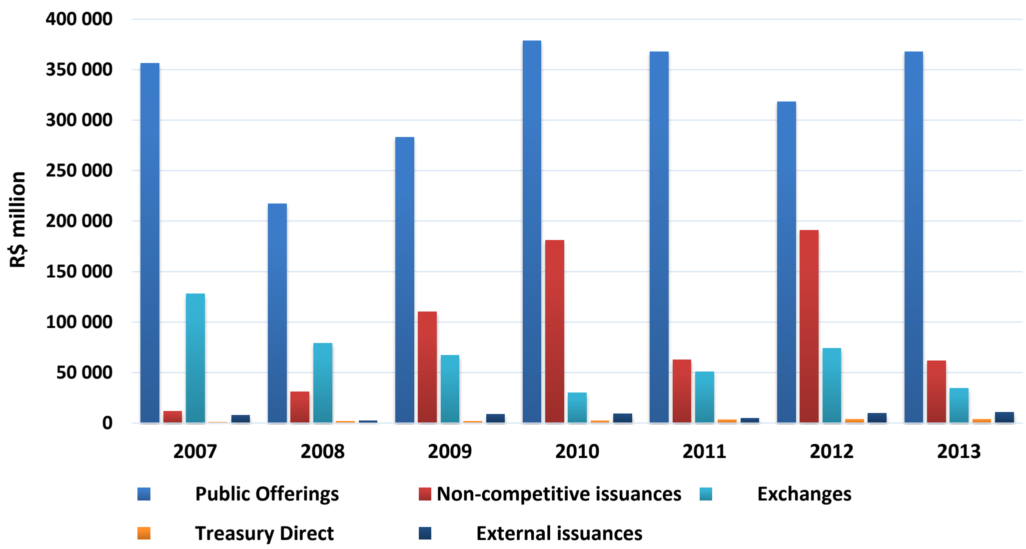
3 BRAZILIAN TREASURY AUCTIONS

3.1 BRAZILIAN TREASURY SECURITIES

Federal public debt management in Brazil is under the responsibility of the Brazilian Treasury since 1988. For some time, this responsibility was shared with the Brazilian Central Bank, which could issue its own securities for monetary policy purposes. Since 2002, however, the Central Bank is no longer allowed to issue its own instruments, and uses Treasury securities to implement monetary policy.

There are a number of different operations with government securities in which the Brazilian Treasury engages: domestic public offerings, noncompetitive issuances, exchange and buy back operations, foreign market issuances, and so on. In this work, we will focus on domestic public offerings for two reasons. First, domestic sale of federal government securities are by far the largest operations undertaken by the Brazilian Treasury (see figure 1). Second, they are carried out through auctions procedures that are widely explored in both theoretical and empirical auctions studies.

Figure 1. Amount of debt issued by the Brazilian Treasury by type of issuance



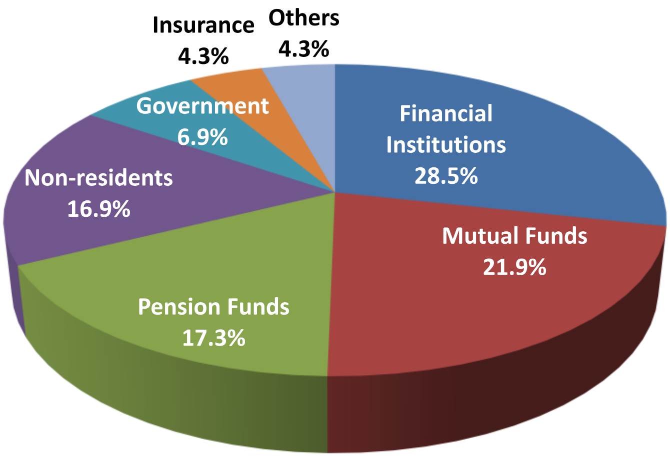
Source: Brazilian Treasury, monthly debt report (December 2013)

In its domestic public offerings, the Brazilian Treasury issues four types of securities: *Letras do Tesouro Nacional* (LTN), *Letras Financeiras do Tesouro* (LFT), *Notas do Tesouro Nacional - série B* (NTN-B) and *Notas do Tesouro Nacional - série*

F (NTN-F). The LTN are fixed-rate Treasury bonds and are issued with maturities of 6 months and 1, 2 and 4 years. The LFT are bonds indexed to overnight interest rates and are usually issued with maturities of 4 years. The NTN-B are inflation-indexed notes issued with maturities of 10, 20, 30 and 40 years. Finally, the NTN-F are fixed rate notes issued with maturities of 5 and 10 years.

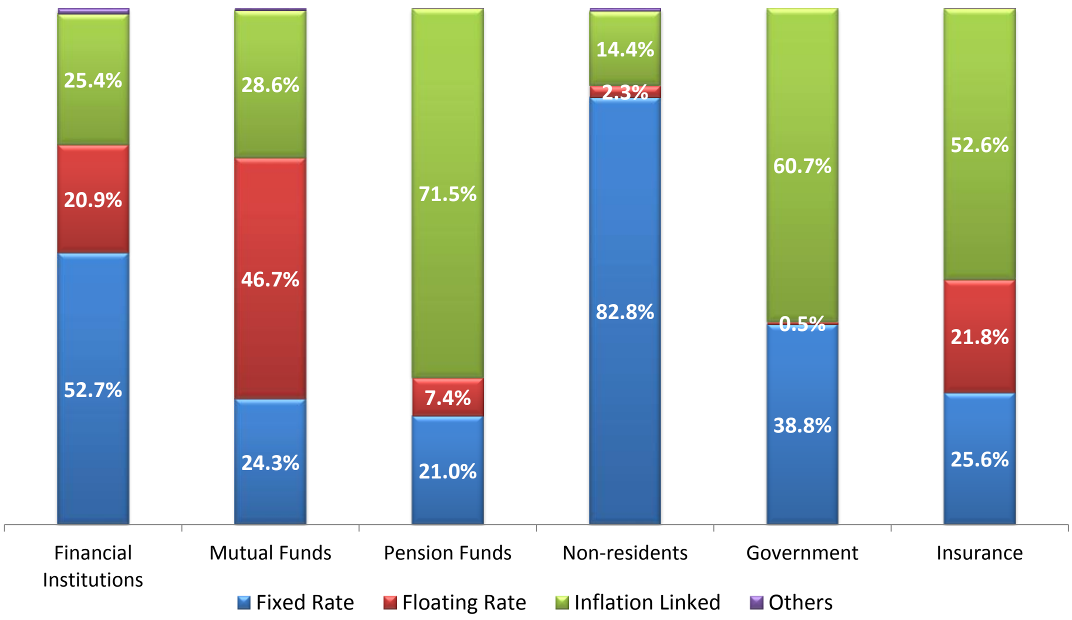
There are several reasons for which an investor may have interest to participate in an auction of government securities. Amongst the most common reasons are meeting liquid-asset reserve requirements, buying and holding the securities and reselling in the secondary market. Because the Brazilian Treasury instruments have so diverse characteristics, different instruments are usually purchased for different purposes.

Figure 2. Brazilian Domestic Debt Holders



Source: Brazilian Treasury, annual debt report (2013)

Figure 3. Profile of Domestic Debt Holders



Source: Brazilian Treasury, annual debt report (2013)

LTN and NTN-F are usually sought by financial institutions and non-resident investors. These investors monitor closely market conditions and seek to derive gains from movements in the term structure of interest rates. Financial institutions may also buy these securities to meet reserve requirements monitored by the Central Bank.

LFTs are mainly carried by banks in their own portfolios, as well as investment funds, as both prefer assets linked to overnight interest rates. The daily liquidity offered by the investment funds, coupled with the daily announcement of the funds' shares, is a key reason for the significant presence of LFTs in their portfolios. These institutions frequently maintain a large part of their resources invested in these bonds to prevent large fluctuation in the quotes, which could induce client withdrawal.

Investors usually demand NTN-B for more than daily liquidity and want to match their liabilities or investment objectives with the bonds' features. These investors are usually buy-and-hold investors and include pension funds and insurance companies. Non-resident investors also participate in this market, contributing to investor's base diversification.

These securities can be purchased in two ways. One is by participating in Brazilian Treasury auctions, according to procedures we are about to describe.

Nevertheless, Treasury securities can also be purchased in the Brazilian secondary market for government securities.

3.2 AUCTION PROCEDURES

Brazilian Treasury auctions are public auctions, open to all institutions registered in the SELIC, a clearing and settlement system managed by the Central Bank, which, among other things, registers the transactions involving government securities. Individuals and institutions not listed in the SELIC can only participate in the auctions by placing orders through one of the listed institutions.

Despite the fact that Brazilian Treasury auctions are open auctions, some participants, known as *primary dealers*, PD, have special rights and obligations regarding these auctions (see Table 3). Currently, there are 12 primary dealers, 10 of which are commercial banks and 2 are brokerage firms or independent distributors. The primary dealers are evaluated biannually based on their performance on primary and secondary public securities markets. Those found unfit are replaced.

Table 5. Primary Dealer's duties and privileges

Privileges
• Primary Dealer status
• Exclusive meetings with the Treasury
• Participation on special operations (non-competitive subscriptions and buyback auctions)
• PDs League tables
• Number of bids in auctions (7 for PDs and 3 for non-PDs)
Duties
• Quote bid and offer prices (secondary market) on electronic trading systems for selected securities
• Have a minimum of 8% of market share on three securities chosen by the dealer from the list of securities established by the Treasury
• Have a minimum participation of 4% of total amount of securities auctioned in the previous month
• Provide market information to the Treasury's operations desk

Source: Brazilian Treasury

Auctions are held weekly through an electronic system managed by the Central Bank. Each regular bidder is allowed to submit up to three quantity-price bids and primary dealers up to seven bids. Bids are submitted electronically and organized in decreasing order of price. The cut-off price is established at the point where demand equals supply[17]. The pricing method differs according to the type of security. The auctions of LTNs and NTN-Fs are discriminatory and every winning bidder pays its own bid. NTN-Bs are sold through uniform-price auctions in which every winning bidder pays the lowest winning bid. LFTs were auctioned under a uniform pricing rule until March 2012, when the Brazilian Treasury switched to a discriminatory format.

Before the auction announcement, the Treasury prospects demand through direct contact with market participants, especially primary dealers. The primary dealers are penalized in their biannual evaluation if the dealer does not place bids in the auction adherent to his previous indication during the demand survey. The Brazilian Treasury utilizes this information to establish the amount that will be auctioned.

The auction announcement is typically done in the same day the auction is carried out and contain all relevant information for the participants, such as types and maturities of securities to be issued, pricing method, maximum amount to be auctioned (supply), deadline to present the bids and so on. Here, we make another distinction. In LTN, NTN-F and LFT announcements, the supply is defined for each maturity. In NTN-B announcements, the supply is defined for all maturities combined and it is left at the Treasury's discretion to choose how this amount will be distributed among the different maturities listed[18].

Bids are presented from 11:00h to 11:30h for LTN, LFT and NTN-F auctions. In the NTN-B auctions, bids are received from 12:00h to 13:00. Bids must respect the quantity multiple of 50 securities and prices must be presented with 4 decimal places for NTN-B and LFT auctions and 6 decimal places for LTN and NTN-F auctions. After bids are received and processed, the Brazilian Treasury determines the cut-off price of each auction. The Treasury also retains the right to reduce the amount to be

[17] The Treasury may alter the announced supply after receiving the bids. Altering the supply will obviously alter the cut-off price.

[18] Rodrigues and Bugarin (2003) analyzed this process, which market participants call a hybrid auction, and confirmed the advantage of the mechanism when there is uncertainty regarding the actual demand for securities.

auctioned or even cancel the auction. The decision to adjust the supply or cancel an auction is usually taken when bad market conditions entail highly dispersed bids or when the Treasury's own valuation is significantly different from what is reflected in the bids received.

After the auction process is completed, the Central Bank, under Treasury request, discloses the auction results. Each bidder is privately informed about the quantity awarded and its price(s). Publicly, the only information disclosed is the cut-off price and the amount actually sold in the auction.

Differently from some of its peers, the Brazilian Treasury does not impose any type of reserve price or rate in its auctions. There is no limit to the amount a single bidder is allowed to acquire, a feature that is also present in some other countries. Noncompetitive bidding exists in a restricted form, the so-called special operations.

Primary dealers have the right (but not the obligation) to engage so-called special operations with the Treasury: each dealer may, after the auction is complete and the result disclosed, purchase a pre-announced amount[19] of securities at the quantity-weighted average price established in the auction. This type of operation only takes place if at least 50% of the auction's pre-announced supply is sold. The special operations are similar to the noncompetitive bidding mechanism described in the literature review. However, two important aspects separate these two mechanisms. Brazilian Treasury special operations are restricted to primary dealers, while noncompetitive bidding are generally open. In addition, the special operations do not reduce the amount available for competitive bidders, as it represent an additional amount available to primary dealers[20].

[19] Limited to 20% of the auction's lot in LTN, NTN-B and NTN-F auctions, and 5% in LFT auctions.

[20] We are aware that the special operations are likely to produce endogenous impacts in equilibrium bidding, In our analysis, however, we overlook this fact because the amount sold in these operations are limited to a small fraction of the auction supply.

3.3 THE DATA

Our data consists of bidder level information on all public offering auctions carried out by the Brazilian National Treasury between January 2010 and August 2013. It encompasses 1249 auctions in which 116 different bidders take part. The complete dataset includes auctions of all four types of securities auctioned by the Brazilian Treasury: 111 LFT auctions, 515 LTN auctions, 385 NTN-B auctions and 238 NTN-F auctions. The summary statistics of the data set reveal some important characteristics of Brazilian Treasury auctions.

Table 6. Summary statistics for the entire data set

	Mean	Std. Dev.	Max	Min
# of bids per auction	16.68	11.01	102	1
# of bidders per auction	8.56	4.47	35	1
# of bids per bidders per auction	1.95	1.27	7	1
Bid to Cover Ratio	123.19%	89.17%	700.76%	0.15%
Supply Adjustment Ratio	65.52%	38.60%	100.00%	0.00%

Source: Brazilian Treasury

Although there are 116 different bidders participating in the auctions, this figure does not imply highly competitive auctions. On average, less than nine bidders participate in these auctions and the maximum number of bidders entering a specific auction is thirty-five. Similar conclusion can be drawn from the bid-to-cover ratio, which we calculate in each auction as the total quantity submitted by the bidders divided by the supply announced by the Treasury. On average, the demand in these auctions surpassed the supply by 23%. Both statistics indicate that the Brazilian Treasury auctions are not highly competitive ones.

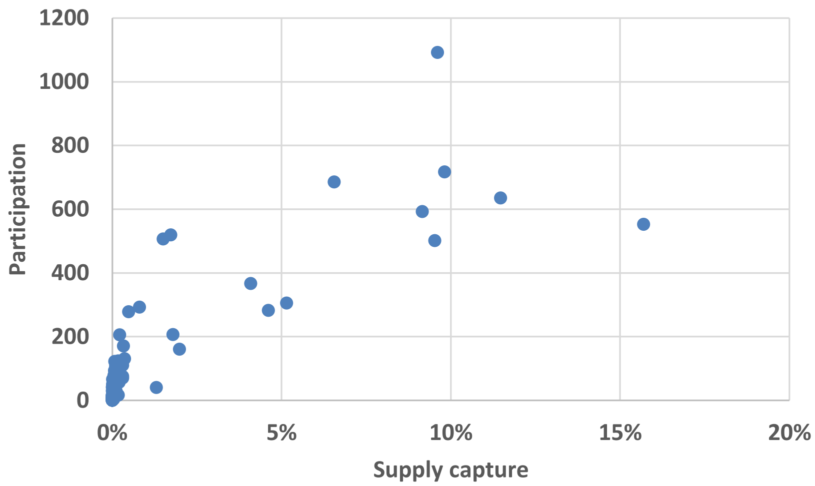
In the previous section, we have learned that the Brazilian Treasury imposes a limit to the number of bids a bidder may submit at any particular auction: three bids for regular bidders and seven bids for primary dealers. Nonetheless, the average number of bids submitted by bidders per auction is less than two. Submitting fewer bids than the maximum allowed appears to be a common behavior in practice (KASTL, 2012), despite the fact that this behavior implies losses for the bidder. Kastl

(2012) shows that the loss of submitting fewer bids is small, which provides a possible explanation for the observed behavior.

We have also learned in the previous section that the Treasury retains the right to reduce the announced amount or even cancel the auction. The supply adjustment ratio, calculated as the actual quantity sold in the auction divided by the auction's announced quantity, shows that the Brazilian Treasury often uses this prerogative. On average, the Brazilian Treasury actually sells approximately 65% of the total quantity previously announced.

Another feature that seems to play an important role in Brazilian Treasury auctions is asymmetry. From the total 116 bidders, a small set of bidders seem to dominate the auctions, both in terms of participation and the percentage they capture from the amount sold at each auction. In fact, the top five bidders captured 56.12% of the total amount sold in these auctions. If we consider the top fifteen bidders, this percentage rises to 94%. The primary dealers alone capture close to 81% of the supply. The pool of bidders can thus be divided into two basic groups. A small group of well-informed bidders, which enter a large number of auctions and are able to capture a large share of the supply. The second group consists of a large number of bidders, which participate in few auctions and either demand a small share of the supply or are unable to submit winning bids frequently.

Figure 4. Participation and supply capture (complete dataset)



Top 5	56.12%
Top 10	85.69%
Top 15	94.00%
Dealers	80.79%

Source: Brazilian Treasury

This initial analysis of our complete data set is useful to observe some general features that characterize the Brazilian Treasury auctions (at least in the period covered in the data). The main objective of this work, however, is to try to determine empirically which auction format, uniform-price or discriminatory, is the best for the Brazilian Treasury auctions, on revenue and efficiency grounds. To accomplish that, we apply the two distinct empirical approaches presented in our literature review: policy experiment and the structural model.

4 POLICY EXPERIMENT

4 POLICY EXPERIMENT

4.1 EMPIRICAL STRATEGY

The basic requirement to carry out the policy experiment approach is that the Treasury auction under analysis has been subject to a format switch. Fortunately, our data contains one such event. Until March 2012, the Treasury auctioned LFT bonds through a uniform-price mechanism, thereafter the Treasury switched to a discriminatory format.

The other types of securities auctioned by the Brazilian Treasury (LTN, NTN-B and NTN-F) did not experience a format switch. Consequently, it is possible to apply the policy experiment approach only to a subset of our complete dataset, namely the 111 LFT auctions. This subset consists of 93 auctions carried out under a uniform pricing mechanism and 18 auctions under the discriminatory format. In terms of their general features, the LFT auctions do not show major differences from the complete data set.

Table 7. Summary statistics for LFT auctions

	Mean	Std. Dev.	Max	Min
# of bids per auction	20.41	18.79	95	4
# of bidders per auction	12.89	7.01	35	2
# of bids per bidders per auction	1.58	1.06	8	1
Bid to Cover Ratio	125.64%	126.15%	690.43%	9.51%
Supply Adjustment Ratio	54.55%	37.19%	100.00%	0.00%

Source: Brazilian Treasury

As we presented earlier, the policy experiment approach is actually a regression analysis of profitability. It consists of calculating the aggregate profit obtained by the bidders in each auction and then regressing the aggregate profit against a set of explanatory variables, which include a dummy for the auction format.

The rationale behind this approach is that the aggregate profits obtained by the bidders in each auction is inversely proportional to the revenue the Treasury is able to extract from the bidders. A positive coefficient in the regression implies a negative effect of that particular variable over the Treasury revenues, and vice versa.

The center of the analysis is thus the coefficient on the dummy variable, which will indicate whether the auction format switch produced any statistically significant effect over bidder profitability (Treasury revenues).

Contrary to what may appear, though, the policy experiment is not a uniform method. The rationale of the method is always the same, but the studies that have explored it so far differed in their choice of regression models, profit measurement and explanatory variables. This is somewhat expected, since each work analyses different auction data sets, in different periods and often in different countries. Anyway, these differences show that the researcher must make some choices (sometimes arbitrary) before employing the policy experiment approach.

Our strategy to tackle this issue is straightforward. We start by using as reference Umlauf (1993)'s work, one of the earliest and most cited work in the policy experiment literature. Then, we check the robustness of the results by varying the econometric treatment applied to the data and replacing the explanatory variables by close proxies and variables used in other works.

Umlauf (1993) applies the policy experiment to Mexican T-Bill auctions, and uses the following regression model:

$$Profits_t = \beta_0 + \beta_1 * Overnight_t + \beta_2 * Bidders_t + \beta_3 * Bidvariance_t + \beta_4 * Quantity_t + \beta_5 * Maximum_t + \beta_6 * Uniform_t + e_t$$

Where *t* indexes auctions and aggregate profits are calculated as:

$$Profit_t = \frac{Weighted\ average\ resale\ price}{Weighted\ average\ auction\ price} - 1$$

Umlauf considers as the resale price, the quantity weighted price of the Wednesday Mexico City money market transactions. At the time his study was realized, Mexican T-Bill auctions were held on Tuesday, their results announced on Wednesday and the settlement took place on Thursday, the same settlement day of the Wednesday money market transactions.

In our analysis, we use a similar measure of bidder profitability. But instead of using average resale and auction prices, we use discount factors, a choice that yields identical results. Another difference is that we consider as the resale price, the quantity weighted discount factor of the transactions in the secondary market[21] at the same day of the auction. The Treasury announces the auction results shortly after the auction procedure is finished and, therefore, the bidders are able to trade the securities purchased at the auction in that same day. Both auction and secondary market transactions are settled in the following working day.

We do not use the exact same set of explanatory variables present in Umlauf's model, for reasons that we shall soon explain. Instead, we use the rationale underlying each variable choice present in Umaluf's work and suggest analogous variables that follow the same rationale. For this reason, we will not describe in detail the construction of the variables used by Umlauf, we will rather concentrate on extracting the rationale behind every variable choice. Furthermore, Umlauf had reasons to consider the existence of a cartel in Mexican T-Bill auctions. Since we have no grounds for similar suspicion, we will omit the rationales related to the possibility of a cartel.

Overnight is the variance of prices of one-month bonds implied by overnight lending rates over the five days leading to the day of auction execution. What Umlauf is trying to capture through this variable is market uncertainty that could disperse bidders' expectation and alter bidding behavior. As a measure of market uncertainty, we use the volatility of the one-month rate of the fixed rate yield curve estimated by an exponentially weighted moving average. We name this variable ***Vol***.

Bidders is the number of bidders participating in each auction and is a measure of competition. In our analysis, we also use the number of bidders as a proxy for level of competition and name it ***Nbidders***. We use the bid-to-cover ratio as an alternative measure of competition (***Bid-to-cover***). We calculate the bid-to-cover ratio in each auction as the total quantity submitted by the bidders divided by the supply announced by the Treasury.

Bidvariance is the weighted variance of winning bid prices and serves as a proxy for information dispersion among bidders. We use a similar proxy for information dispersion, ***Bidvar***, in which we replace prices with discount factors:

[21] Secondary market data is based on the transactions registered in the SELIC system.

$$Bidvar = \frac{\sum_{i=1}^{N}(b_i - b_w)^2 * V_i}{\sum_{i=1}^{N} V_i}$$

where

$b_i = discount\ factor\ of\ the\ ith\ winning\ b_i$

$N = number\ of\ winning\ bids$

$V_i = quantity\ of\ the\ ith\ winning\ bid, and$

$b_w = \frac{\sum_{i=1}^{N} b_i * V_i}{\sum_{i=1}^{N} V_i}$

We also adopt an alternative measure to capture information dispersion, **Bid-ask spread**, which is adopted in other works, such as Goldreich (2007). The bid-ask spread is the difference between the maximum and minimum yields in the secondary market over the day of the auction.

Quantity is the quantity submitted in each auction, and measures the auction size. We adopt the logarithm of the sum of the quantity of the winning bids in each auction, named ***Log(qacc)***. We also use an alternative proxy for auction size, ***Log(supply)***, which is the logarithm of the preannounced supply.

Maximum is the ratio of the pre-specified guaranteed maximum of noncompetitive bids to the aggregate auction quantity. By adopting this variable, Umlauf intends to detect the effect that noncompetitive bids, which reduce auction competitive supply, have on bidder profitability. As we mentioned in section 3, in the Brazilian Treasury auction the noncompetitive bids (known as special operations) do not reduce the auction supply. However, we have also mentioned that the Brazilian Treasury often uses this prerogative of reducing the announced quantity of the auction. For this reason, we replace Maximum for ***Acratio***, the supply adjustment ratio, which is calculated as the auction announced quantity divided by the actual quantity sold in the auction.

Uniform is a dummy that takes on values of one and zero for uniform and discriminatory auctions, respectively. Our dummy for auction format is named ***Type***, and assumes the value zero in uniform-price auctions, and one in discriminatory auctions.

In addition to the variables suggested by Umlauf (1993), we also test the inclusion of additional variables, present in works such as Silva (2002) and Goldreich (2007). ***Log(volsec)*** is the logarithm of the quantity traded on the secondary market in the auction date. ***Volspread*** is an interaction between the variables ***Vol*** and ***Bid-ask spread***. We also use the maturities of the securities auctioned, **Maturity**.

4.2 EMPIRICAL RESULTS

We conduct a regression analysis of auction profitability according to the empirical strategy traced in the previous subsection. As earlier explained, we apply the method on a sample of 111 LFT auctions, held between January 2010 and August 2013 (93 uniform-price auctions and 18 discriminatory auctions). Some of these auctions, however, will be dropped from the original sample due to the lack of appropriate data on secondary market transactions at the auction execution day (26 auctions) or because the auctions were cancelled (17 auctions). Our final sample consists of 68 LFT auctions, of which 55 are uniform-price auctions and 13 are discriminatory auctions.

We begin our analysis with an approach very similar to the one presented in Umlauf (1993). We run ordinary least square regressions, employing Newey-West standard errors specified for an MA(3)[22]. The Breusch-Pagan test rejects the null hypothesis of no heteroscedasticity. The Durbin-Watson statistic falls in the inconclusive region, so we are not able to rule out the presence of autocorrelation. Therefore, the choice of employing Newey-West standard errors seems appropriate at first, since it deals with both heteroscedasticity and auto correlation. In these first two regressions, we use as explanatory variables ***Vol***, ***Nbidders***, ***Bidvar***, ***Log(qacc)***, ***Acratio*** and ***Type***. We use as dependent variable the aggregate profit (***ProfD***) obtained as explained in section four. As a first robustness check, we run the same regression with another measure of aggregate profit (***ProfY***[23]), where we employ the

[22] An MA(3) is specified, following the rule of thumb: $m = 0.75 * T^{1/3}$, where T is the number of observations.

[23] $ProfY = Weighted\ average\ resale\ yield - Weighted\ average\ auction\ yield$

auction and secondary market yields, instead of discount factors. Table 6 summarizes the results.

Table 8. Regression analysis – OLS with Newey-West standard errors

	Dependant variable = ProfD	Dependant variable = ProfY
Vol	38.02473 *	-6.46471 *
	(1.97)	(-1.92)
Nbidders	-0.00021	0.00000
	(-0.27)	(-0.01)
Bidvar	0.97980	-0.17435
	(1.19)	(-1.16)
Log(qacc)	0.02988 ***	-0.00561 ***
	(2.88)	(-2.87)
Acratio	-0.02658 *	0.00534 *
	(-1.73)	(1.91)
Type	0.02963 *	-0.00546 *
	(1.72)	(-1.76)
Intercept	-0.14939 ***	0.02816 ***
	(-2.76)	(2.76)
Observations	68	68
Adj R^2	20.19%	22.06%
Durbin-Watson	dl < 1.437165 < du	dl < 1.403751 < du
Breusch-Pagan	chi2(6) = 39.65	chi2(6) = 44.71

Superscripts *, ** and *** indicate statistical significance at 10%, 5% and 1%, respectively.

The results show that the choice of ***ProfD*** or ***ProfY*** as the dependent variable does not affect the results. Yields are inversely proportional to discount factors (although not through a linear relation) and hence one would expect the two regression to exhibit coefficients of opposite sign. That is precisely what happens and, in addition, the level of significance of the coefficients is the same in both regressions. We will focus on the analysis of the regression that has ***ProfD*** as dependent variable.

The coefficient on ***Vol*** is positive and significant at 10%. It suggests that bidders are risk averse and/or that profit increases when market uncertainty is higher.

The coefficient on ***Nbidders*** is negative and insignificant. The insignificance of ***Nbidders*** may be attributed to the fact that variations in this variable are largely related to the entrance and exit of smaller, less informed, bidders. We have seen that Brazilian Treasury auctions present a group of dominant bidders who participate in most of the auctions and capture a large share of the supply. In this scenario, the entrance or exit of smaller bidders produces little effect over the auction results and winning bidders profitability. Therefore, the insignificance of ***Nbidders*** does not mean that competition has no effect over auction results, but is probably related to the choice of its proxy variable.

The coefficient on **Bidvar** is positive and insignificant. The reason for the insignificance of **Bidvar** is probably similar to what we argued for ***Nbidders***. The variance of bids may be not only a product of information dispersion among bidders, but also a product of the entry of smaller bidders into the auctions.

The coefficient on ***Acratio*** is negative and significant at 10%. This is an unexpected result. A high ***Acratio*** means that the Treasury chose to impose little restriction to the auction supply. Therefore, this variable should have a positive effect on bidder profitability, not the contrary.

The coefficient on ***Log(qacc)*** is positive and significant at 1%. This result implies that in larger auctions bidders are able to achieve larger profits.

The coefficient on ***Type***, the center of our analysis, is positive and significant at 10%. This indicates, although weakly so, that the discriminatory format produces higher aggregate bidder profits and, conversely, lower Treasury revenues. This is the same result as in Umlauf (1993).

Before we take this result seriously and try to speculate on its probable causes, we should test the robustness of the results. As we have argued, the choice of the explanatory variables used in the regression is somewhat arbitrary, as there are other variables that seem to be good proxy candidates. Following the empirical strategy traced in section four, we now run new regressions where we replace some of our variables for close proxies and include a few new ones. Table 7 summarizes the results of these regressions.

Regression (1) is identical to the regression presented in table 6 and is presented in the table as reference for comparison. In regressions (2) to (5) we test alternative proxies for competition (***Bid-to-cover***), information dispersion (***Bidsd*** and ***Bid-ask spread***) and auction size (***Log(supply)***). We also test the inclusion of new variables (***Volspread***, ***Log(volsec)*** and ***Maturity***), that mirror additional variables used in other works, particularly Goldreich (2007) and Silva (2002)

Table 9. Regression analysis – alternative specifications

Dependant variable = ProfD	(1)	(2)	(3)	(4)	(5)
Vol	38.02473 *	34.76460 *	46.42595 ***	46.26170 ***	
	(1.97)	(1.88)	(2.68)	(2.9)	
Nbidders	-0.00021				
	(-0.27)				
Volspread					31.04908 ***
					(2.88)
Bid-to-cover		-0.00538 *	-0.00789 **	-0.00906 ***	-0.00835 **
		(-1.88)	(-2.29)	(-3.04)	(-2.56)
Bidvar	0.97980				
	(1.19)				
Bidsd		0.19787			
		(1.41)			
Bid-ask spread			0.10174 ***	0.10762 ***	-0.58814 **
			(4.79)	(4.6)	(-2.5)
Log(qacc)	0.0298816 ***		0.02733 ***	0.02259 **	0.0165
	(2.88)		(2.69)	(2.06)	(1.49)
Log(volsec)		-0.00268			
		(-1.26)			
Acratio	-0.02658 *				
	(-1.73)				
Log(supply)		0.02086	-0.01129		
		(1.28)	(-0.58)		
Maturity				0.00781 *	0.00759 *
				(1.99)	(1.91)
Type	0.02963 *	0.02769	0.02613	0.02516	0.02504
	(1.72)	(1.49)	(1.39)	(1.5)	(1.56)
Intercept	-0.14940 ***	-0.07285	-0.08250	-0.15947 **	-0.11463 *
	(-2.76)	(-0.76)	(-0.88)	(-2.31)	(-1.68)
Observations	68	68	68	68	68
Adj R^2	20.19%	25.11%	25.68%	28.99%	32.98%

Superscripts *, ** and *** indicate statistical significance at 10%, 5% and 1%, respectively.

Some of the new variables (***Log(volsec)*** and ***Maturity***) do not result in significant coefficients and appear to have no major impact on the results, other than a marginal increase in the model's goodness-of-fit. The alternative proxies, on the other hand, result in more statistically significant results.

Bid-to-cover appear as a better candidate as a proxy for competition than ***Nbidders***. The coefficients on ***Bid-to-cover*** are negative and significant in each regression. The negative sign on the coefficient is expected, since increased competition is likely to restrain bidder's ability to extract profit from the auction. In three, out of four regressions it is used, the coefficient is significant at 5%. In fact, it is reasonable to assume that the entry and exit of smaller uninformed bidders does not largely influences the Bid-to-cover, as it indicates to what extent demand surpasses supply.

The inclusion of ***Bid-ask-spread*** produces more statistical significant coefficients than ***Bidvar*** and ***Bidsd***. In all three regression where ***Bid-ask-spread*** is used as an explanatory variable, its coefficient is significant at 5% and positive.

The main result from this new set of regression concerns the coefficient on ***Type***. In our original regression, the coefficient on ***Type*** was significant at 10%. Yet, in the additional four regression where we test different variables, ***Type*** loses its statistical significance. This means that the original results are not robust and, therefore, the inicial conclusion that the uniform-price format generates greater revenues for the Treasury is temporarily spoiled.

One exercise could deal with the lack of robustness in the previous set of regressions. Up to this point, we have conducted our regressions using OLS with Newey-West standard errors. Under the presence of heteroscedasticity and autocorrelation, OLS coefficients remain consistent and the Newey-West standard errors tackle the problem of biased inference. However, under such circumstances OLS ceases to be efficient. This may be a possible reason for the lack of robustness, since the sample used is relatively small (68 observations). The fact that the coefficients on ***Type*** are positive and stable throughout the regression reinforces this possibility.

In the original regressions, the Durbin-Watson statistics falls in the inconclusive region. For conservatism, we chose to use Newey-West standard errors that accounts for autocorrelation. In this last exercise, we disregard the possible existence of autocorrelation and take into account solely the problem of heteroscedasticity. In this new context, we run Weighted Least Square regressions, which are both consistent and efficient under heteroscedasticity. The choice of explanatory variables remains the same used in the former analysis. Table 8 summarizes the results.

The WLS coefficients closely resemble the OLS ones. No major change in magnitude and no change in sign is observed in the coefficients. The t-statistics, on the other hand, changes considerably. The coefficients on ***Vol***, for instance, that were statistically significant in each OLS regression, are now insignificant.

The most important result in this new set of regressions is that the coefficients on ***Type*** remain positive and gain statistical significance in each regression. The lack of robustness on the OLS results concerning the coefficients on ***Type*** is no longer observed, thus strengthening our initial assessment that the uniform-price format generates greater revenues for the Treasury.

Table 10. Regression analysis – WLS

Dependant variable = ProfD	(1)	(2)	(3)	(4)	(5)
Vol	25.58912	17.45304	27.24500	22.62359	
	(1.48)	(0.47)	(1.63)	(1.48)	
Nbidders	-0.00084				
	(-1.14)				
Volspread					28.56285 ***
					(2.66)
Bid-to-cover		-0.00276	-0.00674 **	-0.00613 **	-0.00984 ***
		(-0.88)	(-2.56)	(-2.67)	(-3.32)
Bidvar	0.88946				
	(1.15)				
Bidsd		0.07781			
		(0.44)			
Bid-ask spread			0.08851 **	0.09037 **	-0.53952 **
			(2.23)	(2.4)	(-2.17)
Log(qacc)	0.01586 *		0.0176	0.00965	0.01894 **
	(1.83)		(1.52)	(1.57)	(2.45)
Log(volsec)		-0.00340			
		(-1.5)			
Acratio	-0.01035				
	(-0.83)				
Log(supply)		0.01239	-0.01844		
		(1.26)	(-1.15)		
Maturity				0.00450	0.00894 *
				(1.57)	(1.86)
Type	0.02900 *	0.03696 **	0.02617 *	0.02970 **	0.01954 **
	(1.69)	(2.99)	(1.68)	(2.04)	(2.1)
Intercept	-0.07105	-0.01159	0.01565	-0.06934 *	-0.13125 **
	(-1.63)	(-0.17)	(0.27)	(-1.75)	(2.6)
Observations	68	68	68	68	68
Adj R^2	4.67%	18.35%	15.39%	17.75%	29.44%

Superscripts *, ** and *** indicate statistical significance at 10%, 5% and 1%, respectively.

In his analysis, Umlauf (1993) also concludes on the revenue superiority of uniform pricing, but states:

It is virtually impossible to determine by examining regression results alone the extent to which the destruction of collusion rather than alleviation of the winner's curse eliminated bidders' profits when uniform pricing was imposed.

Nevertheless, his subsequent arguments seem to favor the thesis that the introduction of a uniform-price mechanism broke down an existing collusion of a small group of large bidders.

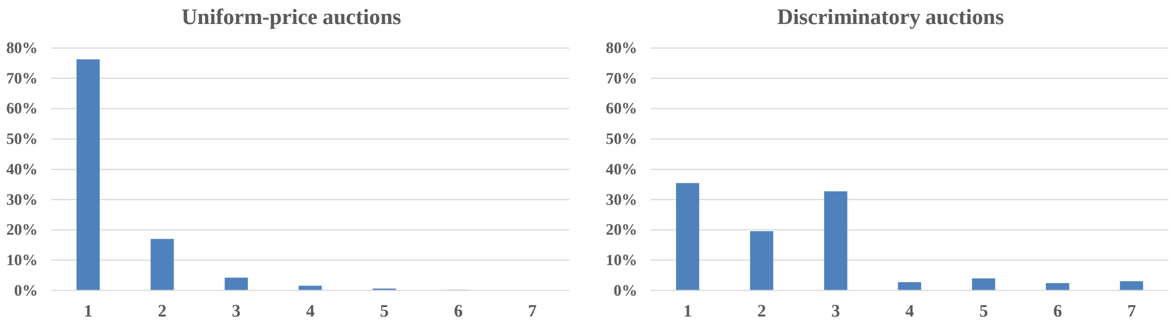
In the present analysis, we are also unable to determine the exact reason that lead to the superiority of the uniform pricing compared to the discriminatory format. Nevertheless, some aspects in bidder behavior indicate that the winner's curse may play an important role.

The frequency with which bidders submit multiple bids suffered a radical change when the auction format switched. Under the uniform-price format, almost 80% of the bids submitted by bidders were single bids. Less than 25% of the bids submitted were in fact multiple bids. When the auction format switched to discriminatory, this scenario changed. Under the discriminatory format, bidders started to use their prerogative to submit multiple bids more often. Indeed, 65% of the bids submitted under the discriminatory format were multiple bids.

At the same time, the share of the supply captured by the top 5, 10 and 15 bidders remained stable under the two distinct formats. Thus, even if collusion is a concern in LFT auctions, the format switch apparently did not affect it. At least not to the extent of altering the capacity of dominant bidders to capture most of the supply.

We therefore suspect it was the strategic response of the bidders, reacting in the face of the winner's curse, which caused bidders to bid more conservatively and thus obtain larger aggregate profits in the discriminatory auctions.

Figure 5. Frequency of bids submitted per bidder per auction



Source: Brazilian Treasury

5 STRUCTURAL MODEL

5 STRUCTURAL MODEL

5.1 EMPIRICAL STRATEGY

To apply the structural model approach to Brazilian Treasury auction's data we follow the techniques developed in Hortaçsu (2002) and expanded in Kang and Puller (2008). These works, especially Hortaçsu (2002), are the most frequently cited in the structural model literature analyzing Treasury auctions and the techniques presented there offer advantages over other structural model approaches found in the literature. Their identification and estimation method is nonparametric and therefore minimizes the impact of distributional assumptions on results. In addition, the method is applied through an empirical algorithm that has low computational demands.

Unlike the policy experiment approach in section 4, in which we analyze LFT auctions, we apply the structural model approach to LTN auctions. The reason for this choice is twofold. First, the LTN bond has been the most important source of financing for the Brazilian Treasury for the past ten years. In 2012, for example, the amount of LTN issued represented 67% of the total amount issued by the Brazilian Treasury in its public offerings of domestic debt. Furthermore, we find that analyzing a different set of auctions enrich our study and enables us to verify if different debt instruments may imply a different revenue ranking.

The complete subset of LTN auctions consists of 515 discriminatory auctions held between January 2010 and August 2013, in which 79 unique bidders took part. In terms of their general features, the LTN auctions do not show major differences from the complete data set.

Table 11. Summary statistics for LTN auctions

	Mean	Std. Dev.	Max	Min
# of bids per auction	16.73	10.86	52	0
# of bidders per auction	7.56	4.10	21	0
# of bids per bidders per auction	2.21	1.34	10	1
Bid to Cover Ratio	153.12%	74.72%	566.67%	0.00%
Supply Adjustment Ratio	86.63%	27.71%	200.00%	0.00%

Source: Brazilian Treasury

5.1.1 HORTAÇSU'S MODELLING

Hortaçsu (2002) models strategic bidding in a divisible goods auctions as an application of Wilson's [1979] share auction model. The share auction model is most easily understood when the bid schedules are modeled as smooth, continuous functions. However, the bid schedules in Treasury auctions are discrete 'step' functions. Therefore, he models bidding in a discrete strategy space in which firms submit a finite number of bid points that are connected with a step function. In this formulation, perfect divisibility of the quantities is maintained but restricted to lie on a discrete price grid. In this subsection we provide a brief description of Hortaçsu's model and his empirical methodology[24].

Let the total supply in an auction be Q with N participating bidders (denoted by $i = 1, \dots, N$) which is assumed to be commonly known to each bidder. Bidders are assumed risk neutral. Let $v_i(.)$ be the true marginal valuation function for bidder i, t_i be the private signal only known to i, and s be a commonly known signal among bidders. The general bidder marginal valuation function is given by $v_i = v_i(q, t_i, s)$ with $v_q \geq 0$ and t_i and s possibly correlated. However, because we restrict valuations to be independent private values, the valuation function is given by: $v_i(q, t_i, s) = v(q, t_i)$. Given the set of possible prices $p_0 < p_1 < \cdots < p_{K+1}$ on a finite grid, each bidder i submits a bid vector y_i defined as quantities specified for each of these prices, i.e.: $y_i = \{y_{i0} \geq y_{i1} \geq \cdots \geq y_{iK+1}\}$.

The market clearing price, p_{k^*}, is determined after sorting all price-quantity bids (in decreasing order of prices) and finding the price at which total demand falls just short of the total supply, i.e.:

$$p_{k^*} \text{is the element on the price grid such that: } k^* = \min\{k\colon \sum_{i=1}^{N} y_{ik} \leq Q\}$$

[24] We try to provide a general view of the model and methodology, but we do not replicate here the proofs and derivations that support the empirical framework. For a complete mathematical description of the model and methodology, we refer to Hortaçsu (2002) and Kang and Puller (2008).

At the market clearing price, p_{k^*}:

$$y_i(p_{k^*}) \cong Q - \sum_{j \neq i}^{N} y_j(p_{k^*})$$

In words, p_{k^*} is the price at which the bid schedule from bidder i intersects residual supply, where residual supply is the aggregate rival bid schedule subtracted from the total quantity supplied.

Hortaçsu defines $H(p_k, y_i)$ as the distribution function of the market-clearing price conditional on submitting the bid vector of bidder i, y_i. Therefore, $H(p_k, y_i)$ *is* the probability that the market clearing price is below p_k conditional on bidder i submitting the bid vector y_i.

$$H(p_k, y_i) = \Pr\{y_{ik} \leq Q - \sum_{j \neq i}^{N} y_j(p_{k^*})\} = \Pr\{\mathrm{p}_{k^*} \leq p_k | y_i\}$$

Given this set up Hortaçsu constructs the expected payoff function of a risk neutral bidder who submits the bid vector y_i

$$\sum_{k=1}^{K} [H(p_k, y_i) - H(p_{k-1}, y_i)] \times \sum_{j=k}^{K} \left(\int_{y_{ij+1}}^{y_{ij}} v(q, s_i) dq - p_j(y_{ij} - y_{ij+1}) \right)$$

The author then derives the first-order condition for the bidder's expected payoff maximization problem and solves for $v_i(y_{ik}, s_i)$, obtaining[25]

$$v_i(y_{ik}, s_i) = p_k + \frac{H(p_{k-1}, y_i)(p_k - p_{k-1})}{H(p_k, y_i) + H(p_{k-1}, y_i)} \qquad (1)$$

---

[25] In one of its appendices, Hortaçsu (2002) investigates this optimality condition more carefully, and concludes that subtle modifications are necessary to account for the cases in which the monotonicity constraint on the bid function is binding. Nevertheless, the author argues that the condition presented above is likely to be correct for most practical purposes.

5.1.2 HORTAÇSU'S (2002) RESAMPLING PROCEDURE

It is evident from the previous expression that if one is able to estimate $H(p_k, y_i)$ and $H(p_{k-1}, y_i)$, it is possible to reconstruct the marginal valuation that rationalizes a bid of p_k for y_{ik} units of bonds. Hortaçsu proposes a resampling procedure to estimate $H(.)$ and make it possible to recover the bidders' marginal valuations.

Kang and Puller (2008) explain the intuition behind the procedure with a didactic example. Suppose that the researcher knew the distribution of bidders' private signals and could compute the equilibrium mapping from signals to bids. The researcher would simulate N draws from the signal distribution, compute the equilibrium bid for each of the N bidders, and find the market-clearing price. By repeating this procedure a large number of times, the researcher could construct the distribution of market-clearing prices.

Since we do not observe data on the signal distribution and the literature does not provide closed-form solutions to equilibrium bids of multi-unit discriminatory auctions, we have to resort to an empirical procedure in order to estimate $H(.)$. That is precisely what the resampling procedure proposed by Hortaçsu does.

The resampling procedure's algorithm in Hortaçsu [2002] is:

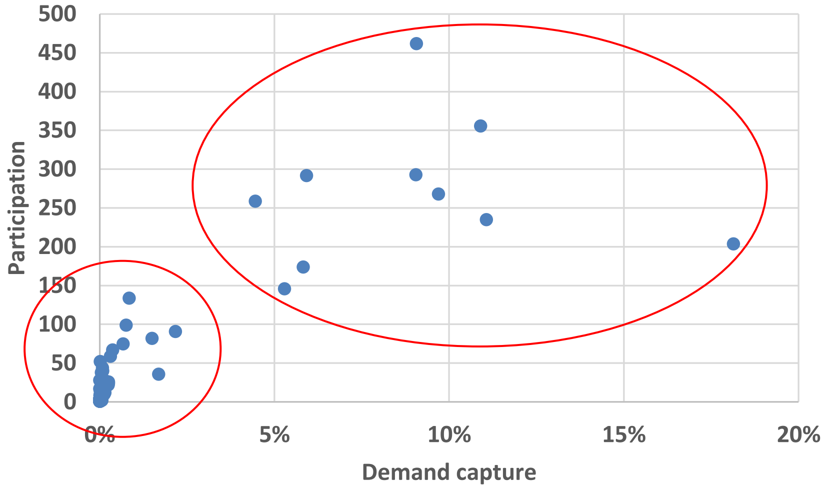
1. Fix bidder i among the N_t bidders in auction t.
2. From the sample of N_t bid vectors in auction t, draw a random sample of $(N_t - 1)$ bid vectors with replacement where a probability $({}^{1}/_{N_t})$ is placed on each vector from the original sample.
3. Using bidder i's observed bid vector and the $(N_t - 1)$ resampled bid vectors, find the market-clearing price where aggregate demand equals total supply. This yields a resampled realization of the market-clearing price, conditional on bidder i's bid vector.
4. Repeat steps 1-3 for each bidder B (a large number) times.
5. Repeat steps 1-4 for each bidder i in auction t.

This procedure generates B market clearing prices for each bidder in each auction, conditional on the bidder's observed bid vector, y_i. Hortaçsu then estimates

$H(p_k, y_i)$ by counting the fraction of draws when the resampled market-clearing price is less than any given p_k. The author labels the above estimator of $H(p_k, y_i)$, $H^R(p_k, y_i)$[26].

The assumption that bidders are ex-ante symmetric has a central role in the resampling procedure presented. However, our initial exploration of the complete data set on section 3 revealed that asymmetry plays an important role in Brazilian Treasury auctions. Similar conclusion is valid when analyzing the subset of LTN auctions. The bidders taking part in LTN auctions appear to be divided in two distinct groups: a small group of dominant bidders, which enter a large number of auctions and are able to capture a large share of the supply, and the remaining non-dominant bidders, which participate in fewer auctions and capture a small share of the supply.

Figure 6. Participation and supply capture (LTN dataset)



Top 5	58.87%
Top 10	89.41%
Top 15	96.37%
Dealers	81.83%

Source: Brazilian Treasury

Fortunately, Hortaçsu's resampling procedure can be adapted to allow for asymmetric bidders. Hortaçsu (2002), Silva (2003a) and Kang and Puller (2008) propose slightly different adjustments to the resampling procedure in order to

[26] Hortaçsu [2002], Proposition 1, part 2 derives conditions for the consistency of the resampling estimator using data from a single auction. It is worth mentioning that the estimator is consistent as N_t goes to infinity.

incorporate asymmetry. We opted to apply the method proposed by Kang and Puller (2008) because of its straightforward nature.

The adjustment to the resampling procedure is simple. In each auction t, we divide the N_t bidders into two groups[27]: a group of N_{1t} dominant bidders and a group of N_{2t} non-dominant bidders. When resampling bid vectors, if i belongs to the dominant group, we draw a random sample of $N_{1t} - 1$ bid vectors where a probability $({}^{1}/{}_{N_{1t}})$ is placed on each vector from the original sample of N_{1t} dominant bidders. We then draw a random sample of N_{2t} bid vectors where a probability $({}^{1}/{}_{N_{2t}})$ is placed on each vector from the original sample of $N2_t$ non-dominant bidders. With these resampled bid vectors $(N_{1t} - 1 + N_{2t})$, we construct the residual supply faced by bidder i and intersect i's actual bid schedule to find market clearing price. If i belongs to the non-dominant group, an analogous operation is realized switching N_1 and N_2. The other steps in the resampling procedure remain the same as the symmetric case.

5.1.3 REVENUE COMPARISON

Once we use the resampling procedure to estimate $H(.)$ for each bidder in any given auction *t*, we are able to apply equation (1) and reconstruct the marginal valuation for each bid vector submitted in that auction. We can then use the reconstructed marginal valuation to compare revenue in the actual discriminatory auction to revenue under a counterfactual uniform-price auction format.

We cannot make a direct revenue comparison between the two auction formats because there are no closed-form solutions for multi-unit auctions that allow us to use valuations to determine the equilibrium bidding under the uniform-price format. We can nevertheless compare revenues in the actual discriminatory auction to revenues in a hypothetic uniform-price auction where every bidder bids truthfully.

[27] We define the dominant bidders as the ones belonging to the group of top ten bidders in terms of supply capture over the entire LTN dataset, as indicated in figure 5. The non-dominant are the remaining bidders.

As mentioned in section 2.3, truthful bidding is an equilibrium strategy in the Vickrey auction, not in the uniform-price format. Therefore, the counterfactual exercise compares the revenue obtained by the Brazilian Treasury in the actual discriminatory auction to the revenue that the Brazilian Treasury would obtain in the uniform-price auction if every bidder bid truthfully. In other words, we compare the actual discriminatory auction revenue with an upper bound for the revenue generated by the uniform-price auction.

5.2 EMPIRICAL RESULTS

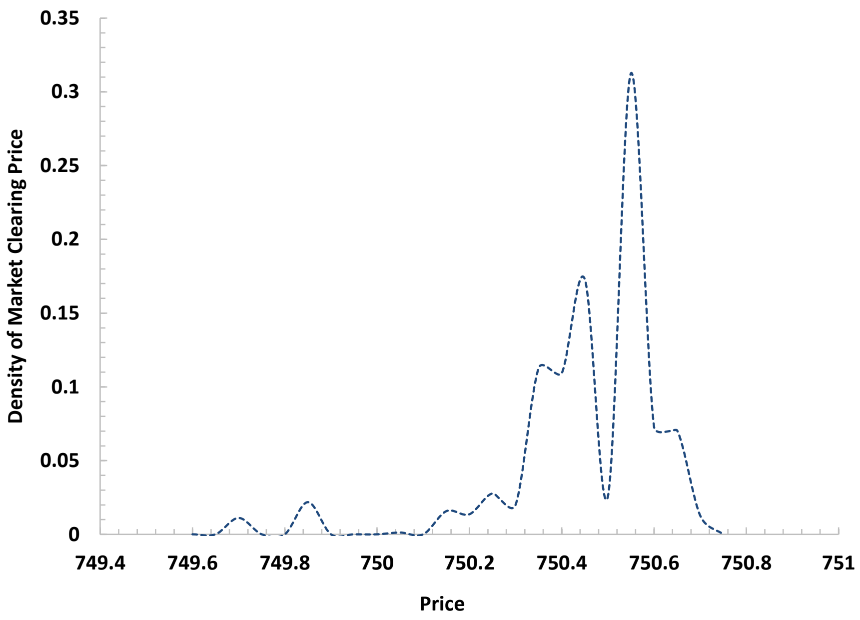
We apply the empirical strategy presented in the previous subsection to our dataset of LTN auctions. Instead of applying the methodology to our entire dataset of 515 discriminatory LTN auctions, which would be computationally time consuming, we chose to apply to the subset of LTN auctions in which at least 15 bidders take part. It is worth to recall that the consistency of the estimator $H^R(p_k, y_i)$ is asymptotic on N_t. Hence, imposing a requirement on the number of bidders entering an auction may be important to guarantee the quality of our results. With this rule, our dataset reduces to 30 LTN discriminatory auctions, a number that is still superior to the number of auctions Hortaçsu (2002) analyzed in his study.

To illustrate the procedure, we show estimates of marginal valuation and the distribution of the market-clearing price for a bidder in a selected auction. As an example, we choose auction #2, held on January 7, 2010, in which 17 bidders (10 dominant bidders and 7 non-dominant ones) competed to purchase LTN bonds amounting to R$ 2.25 billion.

The selected bidder to illustrate the procedure is bidder #20, who submitted five price-quantity pairs in auction #2, the highest price bid being for a quantity comprising 3% of total supply and the lowest priced bid for a quantity comprising 20% of supply. To estimate $H^R(p_k, y_{\#20})$ we hold bidder #20's bid vector constant. Then, we generate a random draw of 16 bid vectors from the sample of 17 bid vectors with replacement, giving equal probability of 1/17 to each bid vector in the original sample. We execute the resampling 10,000 times to generate 10,000 x 16 resampled bid vectors and 10,000 residual supply curves. We intersect these 10,000 residual supply curves with bidder #20's bid function and calculate 10,000 market-clearing prices.

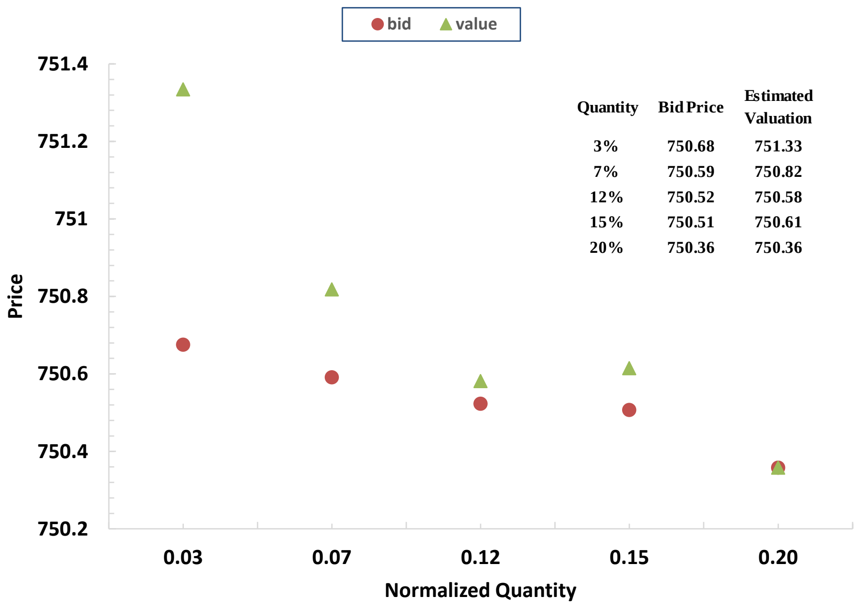
From these 10,000 resampled market clearing prices, we estimate $H^R(p_k, y_{\#20})$ by counting the frequency with which a given price level is above the market clearing price. The density of market-clearing price resulting from this procedure is exhibited in figure 6.

Figure 7. Density of market-clearing price (auction #2, bidder #20)



With the estimated $H^R(p_k, y_{\#20})$, we evaluate the optimality condition (1) with the observed bids to estimate reconstruct the marginal valuation at each bid point.

Figure 8. Actual bids and reconstructed valuation (auction #2, bidder #20)

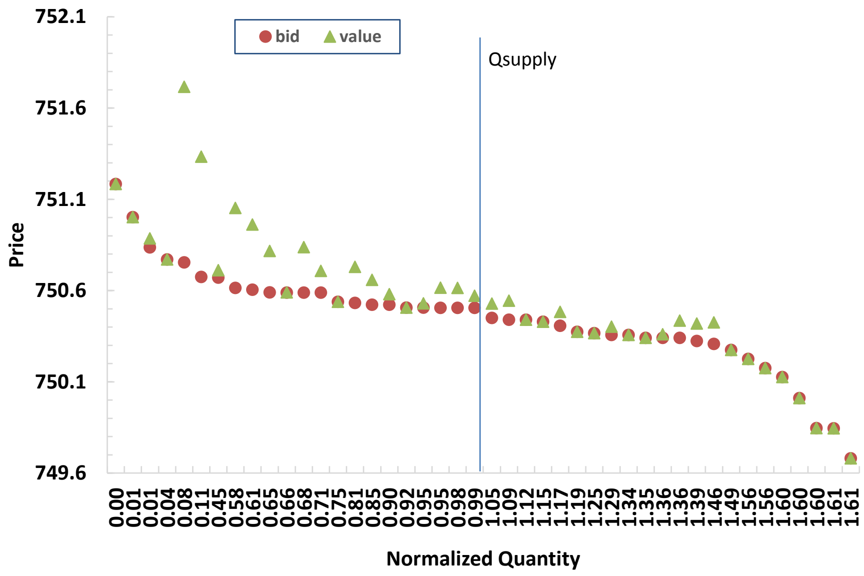


Quantity	Bid Price	Estimated Valuation
3%	750.68	751.33
7%	750.59	750.82
12%	750.52	750.58
15%	750.51	750.61
20%	750.36	750.36

Note: Normalized quantity is the ratio of the bidder's quantity over the total supply

We then repeat the procedure for each of the remaining 16 bidders participating in auction #2 and reconstruct the marginal valuation corresponding to every single bid submitted in the auction.

Figure 9. Aggregate bids and reconstructed valuations (auction #2)



Note: Normalized quantity is the ratio of the bidder's quantity over the total supply

Finally, we use the aggregate valuation to calculate the market-clearing price and the revenue generated under a uniform-price auction in which bidders bid their true value (upper bound revenue of uniform-price auction). Table 10 shows the revenue comparisons after we apply the entire procedure to all thirty auctions[28].

[28] The algorithms were implemented in Visual Basic for Applications (VBA) and are available upon request.

Table 12. Revenue comparison

Auction #	Date	Actual Revenue (R_a, R$ million)	Upper Bound of UP auction (R_u, R$ million)	Revenue difference (%) (= $[R_a - R_u]/R_u$)
2	07/01/2010	2251.86	2251.59	0.0117%
8	28/01/2010	3025.75	3025.47	0.0092%
12	11/02/2010	2296.19	2296.25	-0.0026%
18	04/03/2010	1921.76	1921.64	0.0062%
65	12/08/2010	5671.63	5671.24	0.0068%
73	09/09/2010	3103.43	3102.39	0.0336%
75	16/09/2010	3484.29	3483.64	0.0185%
79	30/09/2010	4656.36	4655.32	0.0224%
81	07/10/2010	4670.97	4670.14	0.0178%
83	14/10/2010	3909.78	3909.56	0.0056%
89	04/11/2010	3537.75	3537.45	0.0085%
93	18/11/2010	1173.67	1173.54	0.0107%
95	25/11/2010	3918.31	3918.22	0.0024%
101	16/12/2010	2294.15	2293.94	0.0090%
106	13/01/2011	3842.98	3842.78	0.0053%
118	10/02/2011	3092.63	3092.13	0.0160%
145	14/04/2011	4597.73	4596.74	0.0216%
146	14/04/2011	1909.86	1909.29	0.0299%
151	28/04/2011	2694.41	2694.36	0.0016%
160	19/05/2011	2723.86	2723.62	0.0089%
166	02/06/2011	3912.39	3912.21	0.0046%
169	09/06/2011	3926.81	3926.57	0.0062%
172	16/06/2011	3933.71	3933.27	0.0113%
181	07/07/2011	2968.76	2968.15	0.0205%
187	21/07/2011	2231.97	2231.77	0.0094%
188	21/07/2011	1647.62	1647.18	0.0271%
229	27/10/2011	3681.61	3680.91	0.0189%
275	16/02/2012	2373.10	2372.63	0.0195%
368	20/09/2012	3019.17	3018.82	0.0115%
371	27/09/2012	2716.46	2716.10	0.0130%

The results of the structural model approach applied to LTN auctions indicate that the discriminatory auction format yields greater revenues for the Treasury, when compared to the uniform-price format. Out of the thirty auctions analyzed, in only one auction (auction #12) the upper bound revenue of the uniform-price auction was superior to the revenue attained in the actual discriminatory auction. The differences in revenue between the two auction formats, however, were quite small. In the twenty-nine auctions where the discriminatory auction turned out revenue superior to the uniform-price auction the average percentage difference was 0.013%. This result is much smaller than the differences obtained by Hortaçsu (2002) using Turkish Treasury auction data, but of magnitude similar to the results produced by Kang and Puller (2008) with Korean Treasury auction data.

The magnitude of the revenue differences raises the question of whether these differences are statistically significant or not. To test the statistical significance of the results we use bootstrapped standard errors to build an *ex-ante* confidence interval. First, we generate 10,000 random resamples of the pair of actual bids and estimated marginal valuations for each auction. In each resample, there are N_t actual bid vectors and N_t marginal valuations vectors drawn randomly from the original set of bids and the set of estimated marginal valuations vectors, respectively. For each resample pair, we compute the market clearing price and revenue under both auctions format and obtain 10,000 values for $[R_a - R_u]/R_u$. We then use the 2.5th and 97.5th percentiles to build our 95% confidence interval. If zero lies within this 95% interval, we cannot reject the null hypothesis $(H_0\colon \frac{[R_a - R_u]}{R_u} = 0)$, but if not, we can conclude that the revenue difference is significantly different from zero. Table 11 reports the results.

In only five auctions, out of the thirty analyzed, we are able to reject the null hypothesis and conclude that the revenue difference is statistically significant. In the five auctions that yielded statistically significant results, the discriminatory format proved revenue superior to the uniform-price format.

In conclusion, the structural model approach applied to LTN auctions suggests that the discriminatory format yields greater revenues than the uniform-price format. The results, however, are both economically small and statistically weak. It is worth recalling that our analysis compared the actual discriminatory format to the “best case” uniform-price format. Therefore, if we could make a more realistic counterfactual comparison it is likely that the revenue differences would increase in both magnitude and statistical significance.

Table 13. Test for Expected Revenue Difference (H_0: $\frac{[R_a - R_u]}{R_u} = 0$)

Auction #	Date	95% Confidence Interval for $[R_a - R_u]/R_u$	Test Result
2	07/01/2010	[-8.343, 0.037]	Not reject
8	28/01/2010	[-16.525, 0.046]	Not reject
12	11/02/2010	[-25.166, 0.042]	Not reject
18	04/03/2010	[-0.006, 0.021]	Not reject
65	12/08/2010	[0.001, 0.02]	Reject
73	09/09/2010	[0.001, 0.057]	Reject
75	16/09/2010	[-12.651, 0.053]	Not reject
79	30/09/2010	[-27.761, 0.034]	Not reject
81	07/10/2010	[-12.015, 0.031]	Not reject
83	14/10/2010	[-16.533, 0.02]	Not reject
89	04/11/2010	[-21.511, 0.023]	Not reject
93	18/11/2010	[0.004, 0.016]	Reject
95	25/11/2010	[-11.682, 0.021]	Not reject
101	16/12/2010	[-24.6, 0.053]	Not reject
106	13/01/2011	[-62.164, 0.013]	Not reject
118	10/02/2011	[-1.569, 0.026]	Not reject
145	14/04/2011	[-38.727, 0.046]	Not reject
146	14/04/2011	[0.005, 0.064]	Reject
151	28/04/2011	[-0.001, 0.013]	Not reject
160	19/05/2011	[-20.824, 0.024]	Not reject
166	02/06/2011	[0, 0.012]	Not reject
169	09/06/2011	[0, 0.029]	Not reject
172	16/06/2011	[-12.192, 0.018]	Not reject
181	07/07/2011	[-31.375, 0.033]	Not reject
187	21/07/2011	[-27.825, 0.015]	Not reject
188	21/07/2011	[0.002, 0.048]	Reject
229	27/10/2011	[-11.112, 0.068]	Not reject
275	16/02/2012	[-26.951, 0.108]	Not reject
368	20/09/2012	[-15.553, 0.034]	Not reject
371	27/09/2012	[-7.264, 0.033]	Not reject

6 CONCLUSION

6 CONCLUSION

This paper analyzed empirical evidence of Brazilian Treasury auction data, aiming at determining which design is more advantageous for the National Treasury, the uniform-price or the discriminatory auction. Although there are other methods that could be used to compare mechanisms and select a specific auction format, in this study we focused on seller's revenue both for its importance as a selection criterion and because there already exists extensive literature exploring this issue.

The debate on the best Treasury auction format is old and its beginning dates back to the 60s. Since then, several researchers have attacked the problem both from a theoretical and from an empirical viewpoint.

In our literature review, we found that part of the early theoretical studies analyzed Treasury auctions through an analogy of single unit auctions or multi-unit auctions with single unit demand and tended to view the uniform-price auction as superior to the discriminatory format. This imperfect analogy, however, fails to capture important strategic aspects of multi-unit auctions with multi-unit demands.

Other theoretical studies accounted for the fact that bidders' ability to submit multiple demands significantly builds up considerable analytical complexity. Although the multi-unit auction theoretical literature has achieved great advances, it has not yet been able to establish a definitive ranking neither in terms of revenue nor allocation efficiency. Multiplicity of equilibria and ambiguity of efficiency and revenues rankings, even under rather simplifying assumptions, seem to be serious obstacles to a definitive policy recommendation.

On the empirical front, the literature divided into two different streams: the policy experiment approach and the structural model approach. These two empirical frameworks have been applied to Treasury auction data of a number different countries and, as occurred in the theoretical literature, also produced ambiguous results in terms of revenues rankings.

We applied both empirical approaches to Brazilian Treasury auction data. We analyzed LFT auctions using the policy experiment framework. The structural model, specifically Hortaçsu's approach, was applied to LTN auctions.

The policy experiment approach applied to LFT auctions revealed that the uniform-price format generates greater revenues than the discriminatory auction. The

WLS regression produced a positive and significant coefficient to the dummy for the auction format. The magnitude of the coefficient, however, shows that the revenue difference found is small. Further explorations indicated that the possible explanation for the revenue superiority of the uniform-price format is connected to bidder behavior when facing the winner's curse.

We obtained opposite results for the structural model approach applied to LTN auctions. The results of the structural model suggested that for LTN auctions, the discriminatory auctions was more advantageous for the Treasury in terms of revenue. Again, the differences were quite small and, in this case, the conclusions lacked proper statistical significance.

This is an important result and shows that the ambiguity in revenue rankings found in the theoretical and empirical literature can also be identified when analyzing data from auctions of different types of securities in a same country.

The small magnitudes in revenue differences between the two auction formats found in this study, as well as in other studies, reveal that the Brazilian Treasury is in a comfortable position to choose its auction format. For practical purposes, both auction formats are close to equivalent in terms of revenue. Therefore, other criteria can be used to determine the auction format.

The ambiguity in revenue rankings and the overall small revenue differences in theoretical and empirical work also seem to point out that the research effort has probably been misplaced. Virtually all theoretical and empirical literature analyze Treasury auctions as a static game, when Treasury auctions are carried out regularly and could be treated as a dynamic (or even repeated) game. Although some authors acknowledge this fact, very few analyzed the dynamic components of Treasury auctions. It is difficult to understand how little attention the dynamic aspects of the Treasury auctions have received in the literature when Friedman's argument, that started the debate, was essentially a dynamic one.

We suggest that future research focus on this aspect and explore features such as the strategic behavior of the Treasury (seller), the dynamic strategic components that determine the bidders' decision of entering or not in the auction, the learning effect on bidders as they play the game repeatedly, the impacts of different primary dealer systems, and so on.

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